

The principle of the mutual energy

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Abstract

Advanced potential solution of Maxwell equations isn't often accepted. We have proven if without advanced potential, it is not possible to satisfy the Maxwell equations. We also shown that it is not the Poynting vector related energy current transferring energy in the space and it is the mutual energy really did that. A important result of the mutual energy theorem is that the advanced potential can suck energy from the transmitter. This energy is equal to the energy received at the receiver. Hence a transmitter can not send any energy out without the receiver. For two remote objects, the energy is transferred only can by the mutual energy of a retarded potential from the source together with an advanced potential from the sink. If the sucked energy is discrete, the summation of mutual energy current of the infinite background atoms or currents, which can be seen as receivers, is a random process. This means that the photon energy sent by the transmitter is actually grabbed by the receiver. Hence the photon from very beginning knows their destination. This receiver send advanced potential to the transmitter. This explanation also avoided the wave function collapse. The retarded potential first reached the receiver, cause the current in the receiver, the current of receiver send a advanced potential to the transmitter with a reversed time, in the same time, a photon minus-time-instantly runs from receiver to transmitter. In our normal feeling, the photon is still runs from the transmitter to the receiver with a positive time. How to transfer superluminal signal using advanced potential is also discussed.

Keywords: Electromagnetic field; Mutual energy; Receive antenna; Transmit antenna; Speed of light; Retarded potential; Advanced potential; Probability; Quantum mechanics; Causality; Wave function collapse, superluminal.

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Part I

Mutual energy current in Fourier space

I. INTRODUCTION

Maxwell equations have two solutions: the retarded potential and the advanced potential. Many physicists accept the advanced potential[39]. The absorb theory of D. T. Pegg, Wheeler and Feynman [41–43] also based on the advanced potential. But most antenna engineers or microwave engineers reject the advanced potential. It is seems without advanced potential the engineer still can solved all engineering problems by using the reciprocity theorem[2–6][15, 16].

The author introduced the concept of mutual energy and the mutual energy theorem[18–20, 40]. There were very closed earlier publications related to the concept of mutual energy [7][9], but they did not realize it is the energy and still thought it as some kind of reciprocity. The author of ref.[18–20] thought this is mutual energy but didn't continue to work with mutual energy for around 30 years. Only recently the authors have found that this mutual energy is so important, it is the only way to transfer energy between two remote objects.

The authors have shown that the reciprocity theorem solution to a system with one transmitter and a receiver is inadequate. This antenna system should be explained by using the mutual energy theorem. In the mutual energy explanation, the receive antenna must send a advanced potential. From the mutual energy theorem, it shows that the advanced potential of the receiver sucks the energy from the transmitter. The transmitter send the retarded potential to the receiver, and then caused the current of the receiver, the receiver send the advanced potential to transmitter, the advanced potential and retarded potential together can transfer the energy form the transmitter to the receiver.

In order to accept the concept of the mutual energy and the advanced potential solution we have to answer following question, if mutual energy transfer the energy, what about the energy current calculated through Poynting vector which is referred as P-current?

If P-current transfer energy also, we found that the advanced potential cannot be accept. Because even in a one dimensional wave guide, there is 4 times P-current for a retarded potential together with advanced potential. Hence we assume that the P-current of the

advanced potential and P-current of the retarded potential all do not exchange energy with other materials in the environment. The concept of P-current is similar to the concept of self-energy in quantum physics and the absorb theory of ref.[41–43].

We have found that in the one dimensional wave guide case, there is a transmitter and a receiver in each ends of the wave guide, the P-current is equal to the M-current which is the energy current calculated by the concept of the mutual energy.

We also shown that in the case of 3D free space, if the the transmitter situated at the center of the free space, the summation of all M-current from the background material to the transmitter is equal to the P-current. Hence in the above two situations the transferred energy still can be calculated through P-current. In a laser beam, the transferred energy can also be approximately calculated through P-current. However in general situation, the calculation with P-current is not the transferred energy. A example is the line antenna with the cross-section area close to 0.

After we accept the advanced potential, and accept the advanced potential can suck the energy from the transmitter, we can discrete the sucking process, hence the background atoms will send discrete advanced potential to the transmitter. There are infinite more this background atoms which can be seen as receivers. Hence the photon energy is randomly sucked out from the transmitter.

The photon from the very beginning knows exactly where it should to reach, this avoid the Schrödinger wave function collapse.

Advanced potential can be seen as ether of the receiver, it can lead the retarded potential, this can offer the reason why the light speed is a constant to the receiver.

The beam width of the mutual energy current is clear different with the P-current. It is easy to obtain the beam section area for the mutual energy current, which has the the maximum value in the middle between the source and sink. This explained the effect referred as antibunching[41–43]. Many results of this paper agree the absorb theory [41–43], i.e., self-energy calculated from P-current can be removed, The spontaneous emission is caused by remote environment materials. However this article offer more details how the advanced potential is cooperate with retarded potential to transfer energy. The retarded potential from transmitter can only cooperate with a returned advanced potential from the receiver and synchronize them together. This cause the energy transfer from transmitter to the receiver.

It is clear this kind of energy transfer can also transfer a electromagnetic force to remote object, a retarded potential send from the transmitter to the receiver, the receiver send advanced potential back, the reaction force on the transmitter is at 0 time. This means at least the electromagnetic force can be reacted at 0 time. If other forces are transferred also similar way with a mutual energy cooperated with a retarded potential and an advanced potential, it can also has a reaction force at 0 time. This perhaps is the reason of for example what is mass, why the action and reaction is equal to each other and can be transferred with 0 time? We also discuss how to transfer a superluminal signal.

II. FOUNDATIONAL THEORY OF ELECTROMAGNETIC FIELDS\\

A. Maxwell Equations

Assume the electromagnetic field system is $\zeta = [E, H, J, K, D, B]$, where E, D are electric field and electric displacement field. H , and B are magnetic fields, H-field and B-field. The magnetic field can be expressed as $\xi = [E, H]$. Assume J and K are current and magnetic current intensities and can be expressed as $\rho = [J, K]$. $\epsilon = \epsilon(t)$ and $\mu = \mu(t)$ are permittivity and permeability which are 3D tensors and a function of time t . ζ is a field system, but we often referred it also as field. ζ satisfies the Maxwell equations,

$$\nabla \times H = J + \partial D \quad (1)$$

$$\nabla \times E = -K - \partial B \quad (2)$$

where $\partial \equiv \frac{\partial}{\partial t}$.

$$D = \epsilon * E, \quad B = \mu * H \quad (3)$$

$\rho = [J, K]$ can be source which sends energy out or sink which receive energy. The example of the source and sink are transmit antenna and receive antenna. In case of scattering, ρ can be the source and sink simultaneously. Another example for ρ is laser sender and laser receiver. $*$ is time convolution where $f * g = \int_{-\infty}^{\infty} f(t)(t - \tau)d\tau$.

B. The superimposition of the fields

Assume in the space there are electromagnetic fields includes advanced potential and retarded potential. The electromagnetic fields can be superimposed. The following is the superimposition of two electromagnetic field systems,

$$\begin{aligned}\zeta &= \zeta_1 + \zeta_2 \\ &= [E_1 + E_2, H_1 + H_2, J_1 + J_2, K_1 + K_2, D_1 + D_2, B_1 + B_2]\end{aligned}\tag{4}$$

Normally there is

$$D_1 + D_2 = \epsilon * (E_1 + E_2)\tag{5}$$

$$B_1 + B_2 = \mu * (H_1 + H_2)\tag{6}$$

One important situation for the superimposition is the case where the two fields one is retarded potential another is advanced potential. This superimposition is not self-explanatory, if we still do not accept the advanced potential. The author accept the advanced potential, and this article we try to make the theory about advanced potential self-consistent.

However the above formula can be generalized as

$$D_1 + D_2 = \epsilon_1 * E_1 + \epsilon_2 * E_2\tag{7}$$

$$B_1 + B_2 = \mu_1 * H_1 + \mu_2 * H_2\tag{8}$$

In the generalized superimposition two fields ζ_1 and ζ_2 are at different space, each space has different media. The authors do not claim that this generalized superimposition is a physics field. The generalized field can be a mathematical field. But if the advanced potential has different media constant, the above superimposition is a physical process. Up to now we still assume that the advanced potential have the same media with retarded potential.

C. The Poynting theorem

Poynting theorem is also be referred as Poynting energy conservation law. There is the Poynting theorem and modified Poynting theorem,

$$\begin{aligned} & -\nabla \cdot (E \times H) \\ & = J \cdot E + K \cdot H + E \cdot \partial D + H \cdot \partial B \end{aligned} \quad (9)$$

where $\partial \equiv \frac{\partial}{\partial t}$. If there is only one media Eq.(5,6), the above formula is referred normal Poynting theorem. If there are different media Eq.(7,8), the above formula is referred as the modified Poynting theorem[40].

D. What will happen if there is no advanced potential in 3D free space?

We have known there is black body which can receive electromagnetic field but do not send the electromagnetic field out. Here we assume it do not send retarded potential out. We also know that the black body received energy by some current. Hence the current in the above black body cannot vanish.

In this time, we assume there exist no advanced potential in nature. We also know that a non-scattering ideal antenna, like a black body, can not send a retarded field out. Hence a ideal received antenna has no any field associated to it. No any field retarded or advanced potentials produced by the current J_a, K_a . J_a, K_a (which doesn't vanish. Since we know from our experience there is current in a receive antenna. The subscript a means the current is a receiver hence "absorb" energy.

We have assume there is no any advanced potential in nature, hence, the electromagnetic field in the space can only be caused by all the retarded potential of the transmitters or transmit antennas, i.e.,

$$\xi = \sum_{i \in I} \xi_{t_i} \quad (10)$$

assuming M is the Maxwell operator. When Maxwell operator acted to the field ξ_{t_i} will obtain the source, i.e., $M\xi_{t_i} = \gamma\rho_{t_i}\gamma$ is a constant. Hence

$$M\xi = \sum_{i \in I} M\xi_{t_i} = \gamma \sum_{i \in I} \rho_{t_i} \quad (11)$$

This means there is not possible to have

$$M\xi = \gamma\rho_a \quad (12)$$

Here is $\rho_a = [J_a, K_a]$ is the current of the receive antenna.

Hence if there is no advanced potential, in the place of the current of a receive antenna, it is not possible to satisfy the Maxwell equation.

This shows ether the Maxwell equation is wrong or there must be the advanced potential. It is not possible to obtained current of received antenna from the retarded potential with Maxwell operator.

Hence, it is better to us to assume that the advanced potential exist in nature. If the advanced potential can not be easily measured, perhaps it can be indirect perceived by the result of the theory of the advanced potential.

E. Retarded and advanced potential

A retarded and an advanced potential potential,

$$\xi_r \propto \iiint_V \frac{\rho(x', t_r)}{|x - x'|} d^3x' \quad \xi_a \propto \iiint_V \frac{\rho(x', t_a)}{|x - x'|} d^3x' \quad (13)$$

Here x and x' are 3D vectors. $x' \in V$.

$$t_r = t - \frac{1}{c}|x - x'|, \quad t_a = t + \frac{1}{c}|x - x'| \quad (14)$$

Here c is speed of light. Assume x is restrict to one dimension, $x > x'$, $\rho(x', t) = \delta(x') \exp(j\omega t)$, this is oscillator located at the origin of the coordinates, we have

$$\xi_r \propto \frac{\exp(j\omega t_r)}{x - x'} \quad \xi_a \propto \frac{\exp(j\omega t_a)}{x - x'} \quad (15)$$

here, $x' = 0$, considering,

$$\exp(j\omega t_r) = \exp(j\omega(t - \frac{1}{c}(x - x'))) = \exp(j(\omega t - kx)) \quad (16)$$

$$\exp(j\omega t_a) = \exp(j\omega(t + \frac{1}{c}(x - x'))) = \exp(j(\omega t + kx)) \quad (17)$$

where $k = \frac{\omega}{c}$, hence,

$$\xi_r \propto \frac{1}{x} \exp(j(\omega t - kx)) \quad \xi_a \propto \frac{1}{x} \exp(j(\omega t + kx)) \quad (18)$$

if we do not consider the fact $\frac{1}{x}$, the retarded and advanced potential can be expressed as,

$$\xi_r \propto \exp(j(\omega t - kx)) \quad \xi_a \propto \exp(j(\omega t + kx)) \quad (19)$$

F. Retarded and advanced potential in loss media

If there is energy loss, we can assume that $k = k_0 - ik_{loss}$, hence there is

$$\begin{aligned} |\xi_r| &\propto |\exp(i(\omega t - (k_0 - ik_{loss})x))| \\ &= |\exp(i(\omega t - k_0 x)) \exp(-k_{loss}x)| = \exp(-k_{loss}x) \end{aligned} \quad (20)$$

For the retarded potential, it is attenuated wave. For the advanced potential there is

$$\begin{aligned} |\xi_a| &\propto |\exp(i(\omega t + (k_0 - ik_{loss})x))| \\ &= |\exp(i(\omega t + k_0 x)) \exp(+k_{loss}x)| = \exp(+k_{loss}x) \end{aligned} \quad (21)$$

It is a ascending wave.

III. REVIEW OF THE THEORY ABOUT THE MUTUAL ENERGY THEOREM

The theory for the mutual energy theorem includes the following a few components,

A. The mutual energy theorem formula

Before the full formula of the mutual energy theorem, there are two early version of it, which can be seen in[7][9]. The formula of the mutual energy theorem can be found [18–20]. The formula [18–20] is in Fourier domain. The corresponding time domain mutual energy theorem can be found in ref.[21].

In Fourier domain the modified mutual energy theorem formula can be written as following[40],

$$(\xi_1, \xi_2)_\Gamma + (\rho_1, \xi_2)_V + (\xi_1, \rho_2)_V = 0 \quad (22)$$

V is the volume contains the current ρ_1 and ρ_2 . Γ is the boundary of the volume V . Γ can be chosen as infinite big sphere. The media have to meet the condition,

$$\epsilon_1^\dagger(\omega) = \epsilon_2(\omega), \quad \mu_1^\dagger(\omega) = \mu_2(\omega) \quad (23)$$

Where “ $\epsilon^\dagger = \epsilon^{*T}$ ”. The superscript* expresses the complex conjugate operator, T is matrix transpose and,

$$(\xi_1, \xi_2)_\Gamma \equiv \iint_\Gamma (E_1 \times H_2^* + E_2^* \times H_1) \hat{n} dS \quad (24)$$

$$(\rho_1, \xi_2)_V \equiv \iiint_V (E_2^* \cdot J_1 + H_2^* \cdot K_1) dV \quad (25)$$

$$(\xi_1, \rho_2)_V \equiv \iiint_V (E_1 \cdot J_2^* + H_1 \cdot K_2^*) dV \quad (26)$$

It is possible that the modified mutual energy theorem is not a physical theorem since the media of the two fields ζ_1 and ζ_2 can be different or at different spaces. If we assume they are the same, i.e., $\epsilon_1 = \epsilon_2$ and $\mu_1 = \mu_2$. There is

$$\epsilon^\dagger(\omega) = \epsilon(\omega), \quad \mu^\dagger(\omega) = \mu(\omega) \quad (27)$$

That is lossless condition. Hence in lossless media the mutual energy theorem (note here, there is no “modified”) is established.

B. The mutual energy theorem can be sub-theorem of the Poynting theorem

We have shown that the modified mutual energy theorem can be derived from the modified Poynting theorem or the Poynting energy conservation law. This is also true if the word modified is removed, i.e., the mutual energy theorem can be also be derived from the Poynting theorem.

It is important that we can show that the mutual energy theorem can be a direct sub-theorem of Poynting energy conservation law instead of deriving it from Maxwell equations. In this way, the mutual energy theorem becomes a energy theorem[40].

C. The mutual energy current of a retarded potential and an advanced potential vanishes in the infinite big sphere

Assume ζ_1 is retarded potential and ζ_2 is advanced potential we can prove that in the free space (where the media is ϵ_0 and μ_0) the surface integral of the mutual energy theorem will vanish at infinite big sphere Γ .

$$\lim_{r \rightarrow \infty} \iint_{\Gamma} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n} dS = 0 \quad (28)$$

Here r is the radio of sphere Γ . Assume

$$\lim_{r \rightarrow \infty} H_1 = \frac{1}{Z} \hat{n} \times E_1 \quad (29)$$

Here \hat{n} is the direction of wave ζ_1 which is retarded potential.

$$\lim_{r \rightarrow \infty} H_2 = \frac{1}{Z} (-\hat{n}) \times E_2 \quad (30)$$

Here $(-\hat{n})$ is the direction of wave ζ_2 which is advanced potential. $Z = \sqrt{\frac{\mu_0}{\epsilon_0}}$, the details can be found in [40].

D. The surface integral in the mutual integral is inner product between two retarded potentials or two advanced potentials

The surface integral in the mutual energy formula $(\xi_1, \xi_2)_\Gamma$ is a inner product [18]. Here we have assumed that that the two fields ξ_1, ξ_2 are two retarded potentials. i.e. the surface integral satisfies following 3 inner product laws,

I. conjugate symmetry,

$$(\xi_1, \xi_2)_\Gamma = (\xi_2, \xi_1)_\Gamma^* \quad (31)$$

II linear,

$$(\xi_1 + \xi_2, \xi_3)_\Gamma = (\xi_1, \xi_3)_\Gamma + (\xi_2, \xi_3)_\Gamma \quad (32)$$

$$(\alpha \xi_1, \xi_2)_\Gamma = \alpha (\xi_1, \xi_2)_\Gamma \quad (33)$$

III Positive-definiteness,

$$(\xi, \xi)_\Gamma \geq 0 \quad (34)$$

$$(\xi, \xi)_\Gamma = 0 \quad \text{iff } x = 0 \quad (35)$$

Here *iff* means if and only if.

If the ξ is advanced potential, the other formulas are all established except Eq.(34) need to updated to

$$(\xi, \xi)_\Gamma \leq 0 \quad (36)$$

If we allow the field ξ_1 and ξ_2 are retarded potential and $\xi = \xi_1 + \xi_2$, also advanced potential, the above inner product laws are still satisfied. Only the formula Eq.(35) does not satisfy. This inner product formula guarantee that we can use the inner product expression $(\xi_1, \xi_2)_\Gamma$ it defines the mutual energy current of surface Γ ,

$$Q_{12} = (\xi_1, \xi_2)_\Gamma = \iint_\Gamma (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n} dS \quad (37)$$

The above discussion tell us the mutual energy current is actually a very good inner product. We will apply thin inter product to simplify the formula.

E. The mirror transform of a retarded potential is advanced potential and vice versa

The mutual energy theorem often works with magnetic mirror transform, it is important to know what will happen after the transform for a retarded potential or a advanced potential. This can be found in [40].

$$m\zeta = [E^*, -H^* - J^*, K^*, \epsilon^*, \mu^*] \quad (38)$$

IV. THE SYSTEM WITH TRANSMIT ANTENNA AND RECEIVE ANTENNA

In the case of two coils, the first coil has a current J , which send energy to the free space, if we put the second coil close to the first coil, the second coil will suck more energy from first coil. In this situation the second coil has influence to the first coil. This is the case where

the two coils close to each other, for example a electric transformator which has two coils: the primary coil and a secondary coil. We know that if we add more load to the secondary coil of a transformator, it can suck more energy from the primary coil.

The two coils can be seen as two antennas. If we assume the two antennas are put very far away, is the second coil (or antenna) still can suck energy from the first coil (or antenna)? Does the received antenna only passive receive energy sent from the transmit antenna or it actively suck the energy from the transmit antenna?, like the the secondary coil in a transformator?

Traditionally we thought the receiver antenna, only passive receive the energy from the the retarded wave of the transmitter. The P-current carries the energy from the transmitter to the receiver. But why when the receive antenna close to the receiver it can affect the transmitter, but when it is far away, this ability is disappeared? According to my understand, it should not disappeared completely but just decreased! This section the authors try to answer this question.

A. The traditional way of the explanation of the antenna system using the reciprocity theorem

Here we review the traditional way of using the reciprocity theorem to explain the antenna system. Assume $\zeta_1 = [E_1, H_1, J_1, K_1, \epsilon, \mu]$, $\zeta_2 = [E_2, H_2, J_2, K_2, \epsilon, \mu]$ are retarded potentials, J_1, K_1 and J_2, K_2 are inside the volume V_1 and V_2 respectively. V_1 and V_2 are inside of the volume V , V, V_1, V_2 have the boundary surface $\Gamma, \Gamma_1, \Gamma_2$. the reciprocity theorem is

$$\begin{aligned} & \oint_{\Gamma} (E_1 \times H_2 - E_2 \times H_1) \cdot \hat{n} dS \\ &= \iiint_V (E_1 \cdot J_2 - H_1 \cdot K_2) - (E_2 \cdot J_1 - H_2 \cdot K_1) dV \end{aligned} \quad (39)$$

Since $\xi_1 = [E_1, H_2]$ and $\xi_2 = [E_2, H_1]$ are retarded potentials, it can be proven that the surface integral vanishes if the surface Γ is chosen as a infinite big sphere, i.e.,

$$\oint_{\Gamma} (E_1 \times H_2 - E_2 \times H_1) \cdot \hat{n} dS = 0 \quad (40)$$

Hence there is

$$\iiint_{V_2} (E_1 \cdot J_2 - H_1 \cdot K_2) dV \quad (41)$$

$$= \iiint_{V_1} (E_2 \cdot J_1 - H_2 \cdot K_1) dV \quad (42)$$

For the reciprocity theorem, the media must be symmetric, i.e.,

$$\epsilon^T = \epsilon, \quad \mu^T = \mu \quad (43)$$

If we assume there are $K_1 = K_2 = 0$, there is,

$$\iiint_{V_2} E_1 \cdot J_2 dV = \iiint_{V_1} E_2 \cdot J_1 dV \quad (44)$$

Assume the volume V_1 and V_2 are cylinders and $V_1 = L_1 S_1$, $V_2 = L_2 S_2$. L_1 S_1 are the length and sectional areas of the cylinders, it is same to V_2 , hence there is,

$$\int_{L_2} E_1 dL \iint_{S_2} J_2 dS = \int_{L_1} E_2 dL \iint_{S_1} J_1 dS \quad (45)$$

$$V_{12} I_2 = V_{21} I_1 \quad (46)$$

The reciprocity theorem is derived for two transmit antennas. Where $V_{12} = \int_{L_2} E_1 dL$ is potential on antenna 2 which is produced by current I_1 . $V_{21} = \int_{L_1} E_2 dL$ is the potential on antenna 1 which is produced by current I_2 . $I_1 = \iint_{S_1} J_1 dS$, $I_2 = \iint_{S_2} J_2 dS$. All books of the electromagnetic field theory claim that the above derivation also suit to the antenna systems with a transmit antenna and a receive antenna. However they did not offer a convinced proof.

The above explanation seems correct, but it contains a mortal problem. The authors do not satisfy that. First the current J_1 and J_2 must be both sources, they both send retarded potentials ξ_1 , ξ_2 , (we have known that if and only if ξ_1 and ξ_2 are both retarded potential, the surface integral of the reciprocity theorem vanishes, otherwise the reciprocity theorem of Eq.(42) is not established). In the case there is a receive antenna, we can shown that the reciprocity theorem is wrong in a very simple situation. In the simple situation we assume that the receive antenna is a black body, it can only receive energy. Hence $\xi_2 = [E_2, H_2]$

which is the field produced by antenna 2 send to space, should vanish (we have assume it is retarded potential). Hence, there is $\iiint_{V_1} (E_2 \cdot J_1 - H_2 \cdot J_1) dV = 0$ and the reciprocity theorem becomes

$$0 = \iiint_{V_2} (E_1 \cdot J_2 - H_1 \cdot K_2) dV \quad (47)$$

If $\iiint_{V_2} (E_1 \cdot J_2 - H_1 \cdot K_2) dV$ does not vanish, the reciprocity theory become

$$0 = \text{something not zero} \quad (48)$$

which is clear wrong. if $\iiint_{V_2} (E_1 \cdot J_2 - H_1 \cdot K_2) dV$ vanishes, the reciprocity theorem tell us

$$0 = 0 \quad (49)$$

which is also meaningless.

The above discussion is also suit to the situation where the antenna 2 is a paraboloid antenna. The energy received by the paraboloid antenna does not send again to the space, but send to a feed source. If the scattering energy send from paraboloid antenna to the free space can be omitted, the paraboloid antenna can be seen as black body. For this kind idea paraboloid antenna used as receive antenna, the reciprocity theorem is not suitable.

Second, we have known that there should be the energy current send from transmit antenna to the receive antenna. The reciprocity theorem can not offer us any information about the energy between the two antennas.

Here we do not claim that the reciprocity theorem is all wrong, the reciprocity theorem is still correct to two transmit antennas, which is not very useful. In the following, we will explain the antenna system with the mutual energy theorem.

B. Explanation of a system with a receive antenna using the mutual energy theorem

1. The result of the mutual energy theorem

Instead of using reciprocity theorem we will apply the mutual energy theorem to the system with two antennas, one is transmit antenna and another is receive antenna.

Assume there is a source J_1, K_1 which is corresponding to the transmit antenna and there is a sink J_2, K_2 which is corresponding to the receive antenna. Normally there is scatter

field send out from the receive antenna, we assume the scattering field is small and can be omitted. The corresponding fields to the current J_1, K_1 and J_2, K_2 are ξ_1 and ξ_2 respectively. The following calculation is in the Fourier domain. According to the modified mutual energy theorem in Fourier domain there is[40],

$$(\xi_1, \xi_2)_\Gamma + (\xi_1, \rho_2)_V + (\rho_1, \xi_2)_V = 0 \quad (50)$$

$$\epsilon_2^\dagger = \epsilon_1, \quad \mu_2^\dagger = \mu_1 \quad (51)$$

Considering the antenna 1 is a transmit antenna, assume $\xi_1 = [E_1, H_1]$ is retarded potential. Considering antenna 2 is a receive antenna, the authors assume $\xi_2 = [E_2, H_2]$ is advanced potential. In the above formula $\epsilon^\dagger = (\epsilon^*)^T$, $\rho_1 = [J_1, K_1]$, $\rho_2 = [J_2, K_2]$, the superscript T is matrix transpose and $*$ is complex conjugate operator. Γ is spherical surface located at infinity. For a physical antenna system, we also require that

$$\epsilon_2 = \epsilon_1 = \epsilon, \quad \mu_2 = \mu_1 = \mu \quad (52)$$

The above formula means the receive antenna and the transmit antenna are at the same space with same media ϵ, μ .

The above two formula from the media can be combined together,

$$\epsilon^\dagger = \epsilon, \quad \mu^\dagger = \mu \quad (53)$$

The above formula indicates that the media must be lossless. With the above media condition, the word “modified” before the word mutual energy can be removed, hence modified mutual energy theorem becomes the mutual energy theorem. Hence, the mutual energy theorem is established in lossless media.

We have proven that in the mutual energy theorem, if ξ_1 and ξ_2 one is retarded potential and another is advanced potential, the surface integral vanishes, i.e.,

$$(\xi_1, \xi_2)_\Gamma = \oint\!\!\!\oint_\Gamma (E_1 \times H_2^* + E_2^* \times H_1^*) \cdot \hat{n} dS = 0 \quad (54)$$

Here Γ is infinite big sphere. \hat{n} is the outward unit vector of surface Γ . Hence we have

$$(\xi_1, \rho_2)_V + (\rho_1, \xi_2)_V = 0 \quad (55)$$

or

$$\iiint_V ((E_1 \cdot J_2^* + H_1 \cdot K_2^*) + (E_2^* \cdot J_1 + H_2^* \cdot K_1)) dV = 0 \quad (56)$$

If we have assumed that the magnetic current does not exist, $K_1 = 0$, $K_2 = 0$, the above formula becomes,

$$\iiint_V (E_1 \cdot J_2^* + E_2^* \cdot J_1) dV = 0 \quad (57)$$

If current J_1, K_1 is only inside V_1 and J_2, K_2 is only inside V_2 , we have,

$$\iiint_{V_2} E_1 \cdot J_2^* dV + \iiint_{V_1} E_2^* \cdot J_1 dV = 0 \quad (58)$$

or

$$- \iiint_{V_2} E_1 \cdot J_2^* dV = \iiint_{V_1} E_2^* \cdot J_1 dV \quad (59)$$

$\iiint_{V_2} E_1 \cdot J_2^* dV$ indicate the energy of the field E_1 act on the current J_2 . This is the energy antenna 2 received. This part of energy is negative (assume we have assumed that the energy contributed to free space is positive, this energy is received by antenna 2, and hence cannot be sent to the free space, hence it is negative). Hence $-\iiint_{V_2} E_1 \cdot J_2^* dV$ is positive.

$\iiint_{V_1} E_2^* \cdot J_1 dV$ is the energy sucked by the field E_2 from the current J_1 . This also tell us that the received energy of antenna 2 sent from antenna 1 is same as the energy sucked from antenna 1 by the advanced potential of the antenna 2.

If advanced potential exist, it can suck energy from the source. This sucked energy later is received by the receive antenna. This is the most important result of the mutual energy theorem to the advanced potential and received antenna. The authors believe this result is correct and will apply this to explain more physics result in next sections.

The above important mutual energy formula Eq.(59) is first obtained in Ref. [9] in 1963, can be converter time-domain, which is

$$- \iiint_{V_2} \int_{-\infty}^{\infty} E_1(t + \tau) \cdot J_2(\tau) d\tau dV = \iiint_{V_1} \int_{-\infty}^{\infty} E_2(t + \tau) \cdot J_1(\tau) d\tau dV \quad (60)$$

This formula is first obtained in ref.[21], If $t = 0$ we can obtain that

$$- \iiint_{V_2} \int_{-\infty}^{\infty} E_1(\tau) \cdot J_2(\tau) d\tau dV = \iiint_{V_1} \int_{-\infty}^{\infty} E_2(\tau) \cdot J_1(\tau) d\tau dV \quad (61)$$

This formula is first obtained by Welch[7] in 1960. From above formula is also clear tell us, that the advanced potential of antenna 2 sucks time*energy from antenna 1 $\iiint_{V_1} \int_{-\infty}^{\infty} E_2(\tau) \cdot J_1(\tau) d\tau dV$ is equal to the received time*energy by antenna 2 $-\iiint_{V_2} \int_{-\infty}^{\infty} E_1(\tau) \cdot J_2(\tau) d\tau dV$. Welch knows that E_2 is advanced potential, but he has not Further explain the meaning of his formula. Welch call his theorem time-domain reciprocity theorem instead of some kind energy theorem, this perhaps obstruct him to thought that the advanced potential can suck energy from the transmitters. The first author of this article obtained the mutual energy theorem[18] in 1987, he also did not notice that actually the advanced potential can suck energy from transmitters. Perhaps this is because the advanced potential is a so strange concept, most physicist and engineer reject this concept, how can think it sucks energy?

Interesting to point out from Welch's formula, that the sucked energy does not happen in particular time point, for example,

$$- \iiint_{V_1} E_1(t) \cdot J_2(t) = \iiint_{V_1} E_2(t) \cdot J_1(t) dV \quad (62)$$

but related a integral with time. Consider the photons has particle frequency, the formula Eq.(59) is in Fourier frequency domain. The mutual energy transfer is based on wave of particular frequency, perhaps is related to the fact that the energy particle which is also based on frequency.

2. Similar result compare to the reciprocity theorem

Applying magnetic mirror transform m to the the variable with subscript 2, i.e.,

$$\zeta_2 = m \zeta_{2m} \quad (63)$$

$$= [E_{2m}^*, -H_{2m}^*, -J_m^*, K_m^*, \epsilon_m^*, \mu_m^*] \quad (64)$$

to the formula Eq.(58), we obtain,

$$\iiint_V (E_1 \cdot (-J_{2m}) + E_{2m} \cdot J_1) = 0 \quad (65)$$

or

$$\iiint_{V_2} E_1 \cdot J_{2m} dV = \iiint_{V_1} E_{2m} \cdot J_1 dV \quad (66)$$

After the transform the media formula becomes,

$$\epsilon_{2m}^T = \epsilon_1, \quad \mu_{2m}^T = \mu_1 \quad (67)$$

$$\epsilon_{2m}^* = \epsilon_1 = \epsilon, \quad \mu_{2m}^* = \mu_1 = \mu \quad (68)$$

For simplification, the subscript $2m$ can be rewritten as 3, the above formula becomes,

$$\iiint_{V_2} E_1 \cdot J_3 dV = \iiint_{V_1} E_3 J_1 dV \quad (69)$$

This formula is close to the the reciprocity theorem Eq.(103). It can be applied to prove that when an antenna is used as the receiver or as the transmitter, will have the same directivity diagram. The media formula can be rewritten as

$$\epsilon_3^T = \epsilon_1, \quad \mu_3^T = \mu_1 \quad (70)$$

$$\epsilon_3^* = \epsilon_1 = \epsilon, \quad \mu_3^* = \mu_1 = \mu \quad (71)$$

Actually the Eq.(69,70) are modified reciprocity theorem. Eq.(71) is caused by lossless media that we have used in mutual energy theorem. Substitute Eq.(71) to Eq.(70), i.e., considering $\epsilon_1 = \epsilon$, $\epsilon_3 = \epsilon^*$, $\mu_1 = \mu$, $\mu_3 = \mu^*$, we still get,

$$\epsilon^\dagger = \epsilon, \quad \mu^\dagger = \mu \quad (72)$$

C. Explanation of a system with a receive antenna using the mutual energy theorem in loss media

1. The result of the mutual energy theorem

Instead of using reciprocity theorem we will apply the mutual energy theorem to the system with two antennas, one is transmit antenna and another is receive antenna. For

receive antenna, there should have the wave toward to them, otherwise it can not receiver energy, all fields send from the transmitter to the receiver is a wave toward the receiver, but we have proven that cannot meet the Maxwell equation close to the receiver antenna. Another wave toward to receiver is its advanced potential. Hence we assume that the transmitter send the retarded potential and the receiver send a advanced potential.

We have proven that in the mutual energy theorem, if ξ_1 and ξ_2 one is retarded potential and another is advanced potential, the surface integral vanishes, i.e., $(\xi_1, \xi_2)_\Gamma = 0$. Here Γ is infinite big sphere. \hat{n} is the outward unit vector of surface Γ . Hence we have

$$Q_{loss} = -(\rho_1, \xi_2)_V - (\xi_1, \rho_2)_V \quad (73)$$

Here $-(\rho_1, \xi_2)_V$ is the emitted energy of ρ_1 , $-(\xi_1, \rho_2)_V$ is the emitted energy of ρ_2 , this energy output have been used in media, hence it equal to Q_{loss} . If we take out the minus sign, $(\rho_1, \xi_2)_V$ is the absorbed energy of ρ_1 , $(\xi_1, \rho_2)_V$ is the absorbed energy of ρ_2 .

$$- \iiint_V ((E_1 \cdot J_2^* + H_1 \cdot K_2^*) + (E_2^* \cdot J_1 + H_2^* \cdot K_1)) dV = Q_{loss} \quad (74)$$

If we have assumed that the magnetic current does not exist, $K_1 = 0$, $K_2 = 0$, the above formula becomes,

$$- \iiint_V (E_1 \cdot J_2^* + E_2^* \cdot J_1) dV = Q_{loss} \quad (75)$$

If current J_1, K_1 is only inside V_1 and J_2, K_2 is only inside V_2 , we have,

$$- \iiint_{V_2} E_1 \cdot J_2^* dV - \iiint_{V_1} E_2^* \cdot J_1 dV = Q_{loss} \quad (76)$$

or

$$- \iiint_{V_1} E_2^* \cdot J_1 dV = \iiint_{V_2} E_1 \cdot J_2^* dV + Q_{loss} \quad (77)$$

If $Q_{loss} = 0$, this is lossless situation, there is,

$$- \iiint_{V_1} E_2^* \cdot J_1 dV = \iiint_{V_2} E_1 \cdot J_2^* dV \quad (78)$$

$-\iiint_{V_1} E_2^* \cdot J_1 dV$ is the energy sucked by the field E_2 from the current J_1 . This is J_1 emitted energy.

$\iiint_{V_2} E_1 \cdot J_2^* dV$ indicate the energy of the field E_1 act on the current J_2 . This is the energy antenna 2 received.

This also tell us that the received energy of antenna 2 sent from antenna 1 plus the media loss energy is same as the energy sucked from antenna 1 by the advanced potential of the antenna 2.

If advanced potential exist, it can suck energy from the source. This sucked energy later is received by the receive antenna or is lost in the media. This is the most important result of the mutual energy theorem to the advanced potential and the received antenna. The authors believe this result is correct and will apply this to explain more physics result in the future.

D. The differences of two methods

The above mutual energy theorem result is similar to the explanation of the reciprocity theorem Eq.(42), however there are fundamental differences, see following.

1. J_3 is not a physical current

1) In the mutual energy theorem, the field ξ_2 of the receive antenna is advanced potential. J_2, K_2 is sink which receive wave energy, it is physical current. The current J_3 is a artificial current or some kind of the effective current. Even the field of ξ_3 is retarded potential, but it is also an artificial field, the actually physical field is still ξ_2 which is advanced potential. In the reciprocity theorem the field of the receive antenna ζ_2 is retarded potential.

2. They are established in different media

The mutual energy theorem functions in lossless media. This can not be changed even after we have applied the magnetic mirror transform. The reciprocity theorem is established in symmetric media. Only if the media with real values for both mutual energy theorem and reciprocity theorem can offer similar results. For example the media $\epsilon = i\epsilon_0$ (where ϵ_0 is the Permittivity in free space) is not lossless media but it is symmetric media. This This kind of media doesn't suit to the derivation of the mutual energy theorem, hence the mutual

energy theorem can not apply to the system with transmit antenna and receive antenna.

3. in loss media, mutual energy theorem wins

The mutual energy theorem (normally when we spoke about mutual energy theorem, in the formula, there is $Q_{loss} = 0$) functions in lossless media. This can not be changed even after we have applied the magnetic mirror transform. The reciprocity theorem is established in symmetric media. Only if the media with real values for both mutual energy theorem and reciprocity theorem can offer similar results. For example the media $\epsilon = i\epsilon_0$ (where ϵ_0 is the Permittivity in free space) is not lossless media but it is symmetric media. This kind of media doesn't suit to the mutual energy theorem (in lossless media). In this situation we can apply the mutual energy with energy loss.

$$-\iiint_{V_1} E_2^* \cdot J_1 dV = \iiint_{V_2} E_1 \cdot J_2^* dV + Q_{loss} \quad (79)$$

Assume $Q_e \equiv -\iiint_{V_1} E_2^* \cdot J_1 dV$ is emission energy, $Q_a = \iiint_{V_2} E_1 \cdot J_2^* dV$ is received or absorbed energy, the above formula can be written as

$$Q_e = Q_a + Q_{loss} \quad (80)$$

That also means,

$$Q_a < Q_e \quad (81)$$

Now we validate if we can obtained the above formula from the calculation of with the retarded potential and advanced potential. Assume the distance from receiver to the transmitter is L . J_1 located at 0 and J_2 located at L . We assume $|J_1| \sim 1$, “ \sim ” means “is close to each other”. Hence

$$|J_1| \sim 1 \quad (82)$$

$$|E_1(0)| \sim 1 \quad (83)$$

$E_1(L)$ is retarded potential hence, at distance L the value decrease,

$$|E_1(L)| \sim \exp(-k_{loss}L) \quad (84)$$

J_2 is at the place of L , and it should influenced by the field $E_1(L)$ hence,

$$|J_2| \sim |E_1(L)| \sim \exp(-k_{loss}L) \quad (85)$$

$E_2(L)$ is produced by the current J_2 hence there is,

$$|E_2^*(L)| \sim |J_2| \propto \exp(-k_{loss}L) \quad (86)$$

E_2 is advanced potential, when it go from the started point L to the destination 0, it should increase the value, because in the loss media, the advanced potential is ascending wave Eq.(21), hence,

$$|E_2^*(0)| \sim E_2^*(L) \exp(k_{loss}L)$$

$$\sim \exp(-k_{loss}L) \exp(k_{loss}L) = 1 \quad (87)$$

$$|Q_e| = \left| \iiint_{V_1} E_2^* \cdot J_1 dV \right| \sim 1 \quad (88)$$

$$\begin{aligned} |Q_a| &= \left| \iiint_{V_2} E_1(L) \cdot J_2^* dV \right| \\ &\sim \exp(-k_{loss}L) \exp(-k_{loss}L) = \exp(-2k_{loss}L) \end{aligned} \quad (89)$$

We can see, it is really we can obtain that $Q_a < Q_e$. That also means the mutual energy theorem with loss media can obtained correct results.

In this loss media, if we also assume it is symmetric, the reciprocity theorem still established. Hence we obtain

$$\iiint_{V_2} E_1 \cdot J_2 dV = \iiint_{V_1} E_2 \cdot J_1 dV \quad (90)$$

That is clear wrong for an antenna system with a transmitter and a receiver, and the media has energy loss, the reciprocity theorem is clear wrong. The formula of the reciprocity theorem itself not wrong, it assume the situation where the two antennas are all transmitters. If the two antennas are all transmitters, we can also evaluate the above formula using the similar method.

Since ξ_1 and ξ_2 are all retarded potentials, hence

$$|J_2| \sim 1 \quad |J_1| \sim 1 \quad (91)$$

$$|E_1(0)| \sim 1 \quad E_2(L) \sim 1 \quad (92)$$

$$|E_1(L)| \sim \exp(-k_{loss}L) \quad |E_2(0)| \sim \exp(-k_{loss}L) \quad (93)$$

$$| \iiint_{V_2} E_1 \cdot J_2 dV | \sim |E_1(L)J_2| \sim \exp(-k_{loss}L) \quad (94)$$

$$| \iiint_{V_2} E_2 \cdot J_1 dV | \sim |E_2(0)J_1| \sim \exp(-k_{loss}L) \quad (95)$$

hence we have

$$| \iiint_{V_2} E_2 \cdot J_1 dV | \sim | \iiint_{V_2} E_1 \cdot J_2 dV | \quad (96)$$

4. The scattering process

In the derivation of the mutual energy we have assumed that there is no scattering field sending out from the receive antenna 2. If there is scattering for antenna 2, We can not obtained Eq.(56) and hence Eq.(69).

Using these 3 differences we can design experiments to further examine which is correct, the two theories, the reciprocity theorem or the mutual energy theorem. We can test the theories in symmetric loss media in which the reciprocity theorem is established, but the mutual energy theorem is not established. In this case if we still obtained the same result of reciprocity for a antenna used as receive antenna and transmit antenna, then the reciprocity is correct, otherwise the mutual energy theorem is correct.

There is no any influence of scattering to the reciprocity theorem. But from derivation of mutual energy theorem we have known that the scattering play a important role. Since paraboloid antenna has big difference of scattering when the receive antenna toward to the transmitter or back against the transmitter. If the directivity pattern of a receive antenna

and a transmit antenna is influenced by the scattering, then the reciprocity theorem is wrong and the mutual energy theorem is corrected to the antenna theory.

E. Calculate the mutual energy current

It is clear there are energy current sent from transmit antenna to the received antenna. It is not possible to get this kind of energy current from reciprocity theorem, but it is possible to get this energy current from the mutual energy theorem.

Since we assume that the receive antenna associate only advanced potential (assume there is no scattering, and the transmit antenna sent only retarded potential. We also know that the mutual energy of a retarded potential and an advanced potential have no pure mutual energy go to the outside of the infinite big sphere, i.e. the surface integral of the mutual energy theorem vanishes. Hence all mutual energy current is stored from transmit antenna to the receive antenna. The authors believe this energy current is just the energy sending from transmit antenna to the receive antenna. In the following we offer the formula to calculate the mutual energy current from the transmit antenna to the receive antenna.

Assume there is a transmit antenna which has the source J_1, K_1 and the corresponding field ξ_1 and a receive antenna which has a sink J_2, K_2 and corresponding field ξ_2 . Assume V_1 is the the volume corresponding to J_1, K_1 and V_2 is the volume corresponding to J_2, K_2 . The Γ_1 is the boundary of V_1 . We can apply the mutual energy theorem to the volume V_1 ,

$$(\xi_1, \xi_2)_{\Gamma_1} + (\rho_1, \xi_2)_{V_1} = 0 \quad (97)$$

or

$$(\xi_1, \xi_2)_{\Gamma_1} = -(\rho_1, \xi_2)_{V_1} \quad (98)$$

or

$$\begin{aligned} & \oint_{\Gamma_1} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n}_1 dS \\ &= - \iiint_{V_1} (J_1 \cdot E_2^* + K_1 \cdot H_2^*) dV \end{aligned} \quad (99)$$

If we apply the mutual energy theorem to the volume V_2 there is

$$(\xi_1, \xi_2)_{\Gamma_2} + (\xi_1, \rho_2)_{V_2} = 0 \quad (100)$$

$$-(\xi_1, \xi_2)_{\Gamma_2} = (\xi_1, \rho_2)_{V_2} \quad (101)$$

or

$$\begin{aligned} & - \iint_{\Gamma_2} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n}_2 dS \\ & = \iiint_{V_2} (E_1 \cdot J_2^* + H_1 \cdot K_2^*) dV \end{aligned} \quad (102)$$

From the mutual energy theorem for volume V , we know that,

$$(\xi_1, \rho_2)_V + (\xi_2, \rho_1)_V = 0 \quad (103)$$

or

$$\begin{aligned} & - \iiint_{V_1} (E_2^* \cdot J_1 + H_2^* \cdot K_1) dV \\ & = \iiint_{V_2} (E_1 \cdot J_2^* + H_1 \cdot K_2^*) dV \end{aligned} \quad (104)$$

where $V_1 \subset V$ and $V_2 \subset V$.

Substitute Eq.(99, 102) to Eq.(104), we obtain,

$$\iint_{\Gamma_1} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n}_1 dS = - \iint_{\Gamma_2} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n}_2 dS$$

The above formula shows the mutual energy current send from the transmit antenna Γ_1 is equal to the the energy current received by the receive antenna on Γ_2 . Actually we can choose any surface between the transmit antenna 1 and the receive antenna 2 to calculate the mutual energy current, which is the energy send from transmit antenna to the received antenna. The energy current sending from transmit antenna to the receive antenna can only be calculated by the mutual energy theorem, it can not be calculated by the reciprocity theorem or Poynting energy theorem. This is also shows the importance of the concept of the mutual energy theorem,

The left side of Eq.(99) and Eq.(102) are the mutual energy current go through transmit antenna 1 to the receive antenna 2. The right side of the above formula is the receive energy of the receive antenna.

We know the real Poynting vector is defined as

$$\begin{aligned}
S &= \text{Re}\{E \times H^*\} \\
&= \frac{1}{2}(E \times H^* + E^* \times H)
\end{aligned} \tag{105}$$

Constant $\frac{1}{2}$ can be omitted, hence we can also write

$$S = E \times H^* + E^* \times H \tag{106}$$

Similar to the above definition of Poynting vector, we can define a mutual energy vector,

$$S_{12} = E_1 \times H_2^* + E_2^* \times H_1 \tag{107}$$

The energy current from the transmit antenna to the receive antenna are

$$Q_{12} \equiv \iint_{\Gamma} S_{12} \cdot \hat{n} d\Gamma = \iint_{\Gamma} (E_1 \times H_2^* + E_2^* \times H_1) \cdot \hat{n} d\Gamma \tag{108}$$

Where Γ is any surface between antenna 1 and antenna 2.

V. IS ADVANCED POTENTIAL JUST THE RETARDED POTENTIAL

Lawrence M. Stephenson[39] point out there is one situation there advanced potential can not avoid, that is one dimensional antenna system. For example there is a wave guide, in the left end there is transmitter and in the right end there is a receiver. Assume there is no any energy loss. Hence the retarded potential of the transmitter is also the advanced potential of the receiver.

In the beginning the author thought this is a good idea, i.e. the advanced potential perhaps is the retarded potential of combination of some retarded potential. The authors built the following antenna theory.

A. Simple antenna system

1. One dimension antenna system

For one dimension antenna system the antenna 1 send wave energy out and the antenna 2 receive the wave energy. Assume there is no any energy loss. Hence all energy send from antenna 1 has been received by antenna 2. It is clear that the field between antenna 1 and antenna 2 is retarded potential for antenna 1 and advanced potential for antenna 2.

2. Simple 3 dimension antenna system

Consider another simple antenna system, there is a transmit antenna, which send wave from the source current $\rho_1 = [J_1, K_1]$ and a receive antenna which is at big sphere and the receiver can receive all electromagnetic waves. The current on the antenna 2 is $[J_{s2}, K_{s2}]$ which is surface current. Assume antenna 2 is super conductor and super magnetic conductor, hence on the surface of antenna 2, the electric field and magnetic field both vanish. the current $[J_{s2}, K_{s2}]$ can be found from the boundary conditions as,

$$J_{s2} = \hat{n} \times (H_1^{out} - H_1^{in}) \quad (109)$$

$$K_{s2} = -\hat{n} \times (E_1^{out} - E_1^{in}) \quad (110)$$

where E_1^{out} is the electric field at outside of the big sphere surface. E_1^{in} is the electric field at inside of the infinite sphere surface. H_1^{out} and H_1^{in} have also the similar meaning. Since the antenna 2 are super conductor and super magnetic conductor, $E_1^{out} = H_1^{out} = 0$. For this kind of antenna system, the field send by antenna 1 is retarded potential ζ_1 , but it is also the advanced potential ζ_2 for antenna 2. \hat{n} is the normal vector of the sphere

$$\zeta_1 = \zeta_2 \quad (111)$$

B. The problem of this antenna theory

There is the antenna theory[39], it supported advanced potential, but claim the advanced potential is just part of the retarded potential.

Lawrence M. Stephenson[39] point out, for the simple antenna system, the field between the transmit antenna and the receive antenna can be seen as retarded potential for transmit antenna and received potential for the receive antenna. Every thing is fine. It seems this example support the existing advanced potential. Lawrence M. Stephenson thought this is an example to support the advanced potential.

But after reflectingly thought, the authors can not agree. The author thought this example actually do not support the existing the advanced potential, if the receive antenna has associated the same field as the transmit antenna.

If the advanced potential exists, even for this simple situation the received antenna should has the field of its own. Assume $\rho_1 = [J_1, K_1]$ is the transmit antenna, $\rho_2 = [J_2, K_2]$ is the receive antenna. The field ξ_1 is the field produced by antenna 1. ξ_2 is the field of received antenna 2 which is advanced potential.

The author believe that the total field of this system should be

$$\xi = \xi_1 + \xi_2 \quad (112)$$

That means the field should be possible to be superimposed. Even for this simple situation, the fields transmit antenna should be possible to be superimposed.

We have mentioned if the field of received antenna can be obtained by a combination of other transmit field, i.e.,

$$\xi_a = \sum_{i=1}^N a_i \xi_{t_i} \quad (113)$$

$$\begin{aligned} M\xi_a &= \sum_{i=1}^N a_i \xi_{t_i} \\ &= \sum_{i=1}^N a_i M\xi_{t_i} \\ &= \sum_{i=1}^N a_i \gamma \rho_{t_i} \end{aligned} \quad (114)$$

ξ_a is the absorbed or received.

Where γ is a constant. The above formula is clear conflict to the fact, we need in the place close to the receive antenna, there should be

$$M\xi_a = \gamma \rho_a \quad (115)$$

Where ρ_a is the current of receiver(or absorber). Hence this kind of antenna theory the field can not satisfy the Maxwell equations.

C. The correction to the above antenna theory

1. The superimposition, should be insistent, i.e.,

Even there are retarded potential ξ_1 and advanced potential ξ_2 , which is produced by transmit antenna source ρ_1 and the receive antenna sink ρ_2 . The field should be the superimposition of the two field

$$\xi = \xi_1 + \xi_2 \quad (116)$$

But we also knows that for the simple situation there is

$$\xi = \xi_1 = \xi_2 \quad (117)$$

How can we make the above two different formulas self-consistent?

2. The mutual energy current is important

The mutual energy current is the energy transfer between the two antenna. The Poynting vector of related energy current can not transfer energy between two antennas

Even in one dimensional wave guide system the above 2 points should also be true. Assume the transmit antenna is at $x = a$, and the receive antenna is at $x = b$, here $a < b$,

$$\xi_2 = \begin{cases} -\xi_1 & x < a \\ \xi_1 & x \geq a, x \leq b \\ -\xi_1 & x > b \end{cases} \quad (118)$$

If we superimpose the field according the traditional way there is

$$\xi = \xi_1 + \xi_2 \quad (119)$$

$$= \begin{cases} 0 & x < a \\ 2\xi_1 & x \geq a, x \leq b \\ 0 & x > b \end{cases} \quad (120)$$

Hence, the energy transfer between two antenna is

$$\begin{aligned}
Q &= \iint_{\Gamma} S \hat{n} d\Gamma \\
&= \iint_{\Gamma} (E_1 + E_2) \times (H_1^* + H_2^*) + (H_1 + H_2) \times (E_1^* + E_2^*) \hat{n} d\Gamma \\
&= 4 \iint_{\Gamma} (E_1 \times H_1^* + H_1^* \times E_1) \hat{n} d\Gamma \\
&= 4 \iint_{\Gamma} S_{11} \hat{n} d\Gamma
\end{aligned} \tag{121}$$

Here S is the Poynting vector. $S_{11} = E_1 \times H_1^* + H_1^* \times E_1$. This can be referred the energy current 4 times difficulty. That means if the advanced potential is normal field, in this one dimension antenna system the transferred energy will be 4 times big than the energy transferring energy calculated with the Poynting vector.

The above result is clear not correct. The explanation for this is that the advanced potential is not a normal field, it is the field perhaps in other implicit space. Hence the above field energy calculation is meaning less. The only important between two antennas is the mutual energy which is the transferred energy. the advanced potential can act to the current of a retarded potential. The retarded potential can act on the current of an advanced field. Hence the formula of mutual energy

$$(\xi_1, \rho_2)_{V_2} + (\rho_1, \xi_2)_{V_1} = 0 \tag{122}$$

is still established. In the above formula the first item $(\xi_1, \rho_2)_{V_2}$ is the retarded potential offered energy to current ρ_2 . The second item $-(\rho_1, \xi_2)_{V_1}$ is corresponding to the sucked energy by the advanced potential from the source current ρ_1 .

$$(\xi_1, \xi_2)_{\Gamma_1} + (\rho_1, \xi_2)_{V_1} = 0 \tag{123}$$

The first item is the mutual current send out to the transmitter antenna, second item $-(\rho_1, \xi_2)_{V_1}$ is sucked energy from the transmitter antenna. And we have,

$$(\xi_1, \rho_2)_{V_2} = -(\xi_1, \xi_2)_{\Gamma_2} \tag{124}$$

or

The right of the above formula is the mutual energy send to the receive antenna. The left is the energy current cent to the receive antenna.

The energy transferred from transmit antenna to the received antenna is still the mutual energy current,

$$\begin{aligned}
Q_{12} &= (\xi_1, \xi_2)_\Gamma \\
&= \iint_\Gamma (E_1 \times H_2^* + E_2^* \times E_1) \hat{n} d\Gamma \\
&= \iint_\Gamma S_{12} \hat{n} d\Gamma
\end{aligned} \tag{125}$$

Hence, we can define mutual energy current vector,

$$S_{12} = E_1 \times H_2^* + E_2^* \times E_1 \tag{126}$$

Here, if Γ is chosen between the two ends, since $\xi_1 = \xi_2$ (notice, only the value is same, actually one is retarded potential and one is advanced potential, they are different things), the mutual energy

$$\begin{aligned}
Q_{12} &= \iint_\Gamma (E_1 \times H_1^* + E_1^* \times E_1) \hat{n} d\Gamma \\
&= \iint_\Gamma S_{11} \hat{n} d\Gamma = Q_{11}
\end{aligned} \tag{127}$$

Here, S_{11} is the Poynting vector corresponding the retarded potential ξ_1 . It is same there is

$$Q_{12} = Q_{22} \tag{128}$$

Hence, there is

$$Q_{11} = Q_{12} = Q_{22} \tag{129}$$

In the above the mutual energy Q_{12} is the real energy transferred from the transmit antenna to the receive antenna. This energy in this situation just equal to the energy send out from the transmitter Q_{11} and also the energy received by the receive antenna Q_{22} .

If Γ is chosen less the left end, ξ_1 is the wave toward out side ξ_2 is the wave toward inside, it is clear the mutual energy of

$$Q_{12}(x < a) = 0 \tag{130}$$

If Γ is chosen at outside right end, ξ_1 is the wave toward out side ξ_2 is the wave toward inside, it is clear the mutual energy of

$$Q_{12}(x > b) = 0 \quad (131)$$

From above discussion, in this situation the transferred energy is equal to Q_{12} , the value of Q_{12} is equal to Q_{11} or Q_{22} accidentally. Here we speak about “accidentally” is because $Q_{12} = Q_{11} = Q_{22}$ is only in this simple one dimension wave guide antenna system. In general, $Q_{12} \neq Q_{11} \neq Q_{22}$

D. Summary

If we assume there is no advanced potential in 3D space, the Maxwell equation cannot be satisfied at the place of the receive antenna. Hence there should exist the advanced potential.

If advanced field is the normal field, in the one dimensional wave guide situation, the field between the transmitter and the receiver receiver will be doubled according to the fields superimposition. If the superimposed field is doubled, the energy current will 4 times bigger than the energy current calculated through Poynting vector. This means also that some thing is wrong. Hence we have to assume the advanced potential is not a normal field. Hence the calculation result about the superimposition of advanced field and a retarded field is also not normal. We can assume the advanced potential is at some other space outside our space. However it can act on to current which created the retarded potential.

The advanced potential can suck energy from the current sours (or transmit antenna or transmitter). The sucked energy is equal to the energy of the retarded potential applied to the sink (the received antenna or the receiver). The energy transfer in the space is through the mutual current.

The important thing about energy transfer to an antenna system is the mutual energy current, which is the energy transferred from the transmitter antenna to the receiver antenna.

The energy current calculated through Poynting vector does not offer the energy transferring between transmit antenna and the receive antenna. This is clear if considered a section area of receive antenna wire is close to 0. This kind antenna obtained 0 energy from energy current related to the Poynting vector. However we know that the antenna received energy

is not depending to the section area of the wire. Even the wire is as thin as possible, it still can receive nearly same energy compared to a wire with no zero section area.

The current calculated by Poynting vector is also not the energy transfer between the transmitter and the receiver in the case of one dimensional wave guide. However accidently the mutual energy current is just equal to the energy current calculated by the Poynting vector.

We also answered the question whether the advanced potential can be a combination of other retarded potential, the answer is negative. The advanced potential can not be any combination of the retarded potential, it is the potential “sent” out by the sink current. Hence “sent” is very important, the advanced potential is caused by the sink!

In case of one dimensional wave guide, or 3D situation the background is seen as receiver antenna, the advanced potential is equal to the retarded potential in mathematics values, but they are still different things. But in general situation, for example there is a transmit antenna and a receiver antenna in free space, the advanced potential is not equal to the retarded potential, because the retarded potential is sent by transmitter and the advanced potential is sent from the receiver.

VI. RE-EXPLANATION OF THE ENERGY CALCULATED FROM POYNTING VECTOR

A. The problems of Poynting vector

We have few times met the problems with the energy current calculated from Poynting vector. In the following it is referred as P-current, defined as

$$Q = \iint_{\Gamma} S \cdot \hat{n} d\Gamma \quad (132)$$

where

$$S = E \times H^* + E^* \times H = 2Re\{E \times H^*\} \quad (133)$$

$$Q = (\xi, \xi) = \iint_{\Gamma} S \cdot \hat{n} d\Gamma \quad (134)$$

In our definition of Poynting vector is a real value. We have added a constant 2 to make it has similar definition with mutual energy current density vector, which is defined as

$$S_{12} = E_1 \times H_2^* + E_2^* \times H_1 \quad (135)$$

We can also define the mutual energy current (or for simple M-current) as

$$Q_{12} = (\xi_1, \xi_2)_\Gamma = \iint_\Gamma S_{12} \cdot \hat{n} d\Gamma \quad (136)$$

1. *Receive antenna with thin wire*

First we have known that the received energy from a received antenna is not related to P-current. When the section area of wires become as small as possible the antenna can still receive enough energy. Here we can assume this wire is produced by superconductor material, hence the current on the thin wire can be big enough. This problem is known for long time, it has been also noticed by Ref.[39]. The electronic engineer just overlook this phenomenon.

2. *4-times of the energy current.*

Second in the last section there is so called 4-times difficulty. In one dimensional wave guide or 3D space the transmitter situated at the center of a sphere, the receive antenna is at the sphere, and assume there is no energy loss in the system. In this situation the retarded potential and advanced potential is equal in values. The P-current of superimposed field of two fields in which one is retarded and the other one is advanced, obtains 4-times of the P-current compare to the P-current of the transmit antenna or the receive antenna. This means if we accept the advanced potential, we have to face the 4-time energy current difficulty. Most of us meet this difficulty perhaps will reject the advanced potential. But the authors strongly insist the advanced potential, hence the wrong should be in the side of Poynting vector.

3. *An advanced potential in 3D free space*

We assume there is only a current situated in the center of space, there are no other material. Assume this a current (sink) has advanced potential, the P-current is clear no

zero, that means there is an energy current always toward to the current. We have said the advanced potential only can be used to receive energy. Now it seems even without other source, a sink can still obtained energy, this is very strange. This is also another reason perhaps why normally the people reject the advanced potential.

The authors strongly believe the advanced potential. In order to solve these problems, the authors have to assume that it is not the P-current transfer the energy from a transmitter to a receiver. P-current has no any power to transfer the energy, it is the self-energy current belong the current itself. It does not communicate with other world. The energy current transferring from transmitter to a receiver can only be done with the mutual energy current, i.e., M-current.

In case of one dimensional wave guide situation discussed in last section we have found that the P-current is equal to the M-current accidentally.

In another case, a transmitter antenna sits at the center of free space. The background of free space can be seen as infinite more receiver. Each receivers will send an advanced potential to the transmit antenna. The summation of this all mutual energy current can be calculated by assume the background as big receive antenna, we can show this summation of the M-current from the transmit antenna to the background receiver is just same to the P-current.

We can assume the background as a big antenna situated at the infinite big sphere, this antenna receive all energy from the transmit antenna. Hence it send the advanced field which is same as the field of transmit antenna, between the transmit antenna and the infinite big sphere. That means,

$$\xi_r = \xi_a \quad (137)$$

The P-current is

$$(\xi_r, \xi_r) = \iint_{\Gamma} (E_r \times H_r^* + E_r^* \times H_r) \hat{n} d\Gamma \quad (138)$$

$$(\xi_r, \xi_a) = \iint_{\Gamma} (E_r \times H_a^* + E_r^* \times H_a) \hat{n} d\Gamma \quad (139)$$

Hence we have

$$(\xi_r, \xi_a) = (\xi_r, \xi_r) \quad (140)$$

After this re-explanation for P-current, we can avoid the 4-times difficulty mentioned before and also can explain the reason why a receive antenna received energy is nothing to do with P-current.

After this re-explanation for P-currents, we also do not need to assume the advanced potential is at other space instead of our space, which is also too strange. But any way the retarded potential and advanced potential are still different and they need different method to measure them.

After this re-explanation we also solved the problem of conflict between assumptions of following two formula,

$$\xi = \xi_1 + \xi_2 \quad (141)$$

$$\xi = \xi_1 = \xi_2 \quad (142)$$

where ξ_1 is retarded potential and ξ_2 is an advanced potential. The two formula are all correct. The first is correct in general and ξ is superimposed field, this field perhaps is not a normal field. The second formula Eq.(142) is correct in case of one dimensional wave guide, or in a free space the background can be seen as receive antennas, without energy loss and reflection, and here ξ here is a normal field we can detected with normal technology. In the one dimensional wave guide, here $\xi_1 = \xi_2$ is only on mathematical value, ξ_1, ξ_2 are still two different physical thing ξ_1 is retarded potential, ξ_2 is advanced potential. $\xi = \xi_1$ is tell us in the wave guide the normal field just the retarded potential.

In this re-explanation the P-current of advanced potential do not cause any energy current, hence advanced potential can be existed to arbitrary current. If there is no source or transmit antenna, there is no energy current of advanced potential go towards the current of the receiver. We know that From Maxwell equations we can obtain two solutions one is retarded potential one is advanced potential. Now we know that the advanced potential exists to any current even it is a transmit antenna. The P-current of this advanced potential do not cause an energy toward the current source or sink.

The retarded potential send out the energy, however we have shown it is not the retarded potential's P-current send energy out, it is the back ground which can be seen as infinite small antennas, which have sucked the energy from the transmit antenna through their advanced potential. The summation of all mutual energy current, i.e. M-current just equal to the P-current of the retarded potential.

B. The mutual energy of two retarded potential

Assume there are two current sources $\rho_1 = [J_1, K_1]$, $\rho_2 = [J_2, K_2]$, we have produced two retarded potential, ξ_1 ξ_2 ,

$$\xi = \xi_1 + \xi_2 \quad (143)$$

is also retarded potential, the Poyning theorem can be obtained by

$$\begin{aligned} Q &= (\xi, \xi)_T \\ &= (\xi_1, \xi_1)_T + (\xi_1, \xi_2)_T + (\xi_2, \xi_1)_T + (\xi_2, \xi_2)_T \\ &= Q_{11} + Q_{12} + Q_{21} + Q_{22} \end{aligned} \quad (144)$$

We have known that Q_{11} and Q_{22} and Q are the P-current of ξ_1, ξ_2 and ξ respectively, we have know that this current belong self energy current, it has no any contribution to outside world. $Q_{11} + Q_{12}$ is some part of ξ . It should also no any contribution to the energy change. The energy change of Q_{12} and Q_{21} are also retarded energy current, it will have no contribution to the energy current.

C. Summary about P-current

1. For a current in free space

The authors assume, any currents have advanced potential and retarded potential. P-current of the advanced potential and the retarded potential do not transfer energy. We can also think that the retarded potential it transfer energy to infinite far away and then this energy change to advanced potential and has been received by this current, so there is no any pure exchange of energy to other current. But this need in the infinite remote distance there should have a tunnel which has the power to connect the future to the past.

Since we cannot detector this kind of P-current, that means not only the P-current of advanced potential but also the P-current of the retarded potential all can not be detected.

2. *M-current*

The authors assume, it is only the mutual energy current M-current, which is responsible for the transferring the energy from transmit antenna to the receive antenna. Here the mutual energy of a retarded potential and an advanced potential. The advanced mutual energy must send from the receiver and cause the the current in the receiver changed, and the change a current in receiver send a back a advanced potential to the receiver. Since this this two potential can be synchronized. The mutual energy can send from the transmitter to the receiver in a retarded meaning.

It is also possible the receiver send a advanced potential to the transmitter and the transmitter caused a current change which in turn produced a retarded potential, that potential is send bake to the receiver. In this case the energy is also retarded from the transmitter transferred to the receiver in retarded meaning.

3. *One dimensional wave guide situation*

In case of one dimensional wave guide or the case the transmit antenna in the center of a big sphere, the receive antenna is at the big sphere, the M-current is just equal to the P-current of receive antenna or the P-current of the transmit antenna. Hence we still can calculate the transferred energy by use P-current for this special situation. This waym the calculation result is correct, but the method is wrong.

4. *In case there are of laser beam*

In the case of laser beam if we can calculate the P-current of the receiver, which is

$$Q = (\xi_1, \xi_1) = \iint_{\Gamma_S} (E_1 \times H_1 + E_1^* \times H_1) \cdot \hat{n} d\Gamma \quad (145)$$

and we can also calculate the total M-current, Here Γ_M is the beam area of M-current,

$$Q_{12} = (\xi_1, \xi_2) = \iint_{\Gamma_M} (E_1 \times H_2 + E_2^* \times H_1) \cdot \hat{n} d\Gamma \quad (146)$$

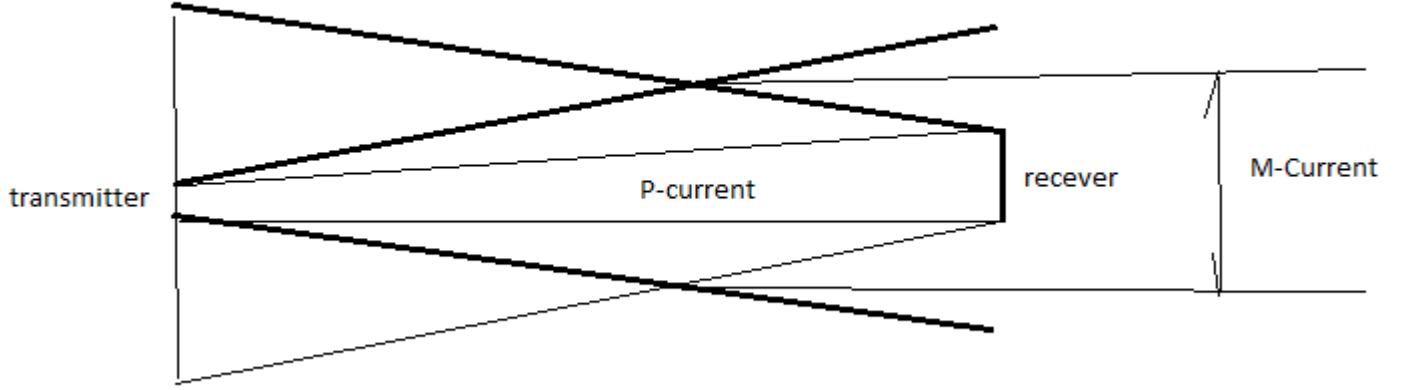


Figure 1: Laser beam situation.

Here Γ_M is the beam area of M-Current. The beam shape for P-current and M-current can be seen in the figure 1.

I believe that the M-current of the receiver is little bigger than the P-current of the transmitter (please see, not only part of P-current has been received by the receiver), because the beam area of M-current is bigger than that of P-current, see Figure 1.

This is also clear for us, we know that line antenna have much bigger effect area than the actual section area of the antenna. The plate antenna has also larger effective receive area than the plate is. The reason is also because energy is transferred by M-current instead of P-current. This also explained as the antibunching[41–43] phenomenon.

VII. THE PRINCIPLE OF THE MUTUAL ENERGY

Normally we speak that Poynting theorem is the energy conservation law for electromagnetic field theory. However summary of this article are much different. The Poynting vector loss some of importance. The concept mutual energy win more importance. The following can be seen as mutual energy principle (because we have not proven it completely, it is accept as principle or law). Assume there is a system with currents (sours and sink) separate remotely.

A. Energy is not transferred by Poynting vector

Assume we have two current ρ_1 and ρ_2 not very close to the origin of the infinite bigger sphere. The current ρ_1, ρ_2 can be seen as remote to each other, they send advanced potential and retarded potential, $\xi_{1a}, \xi_{1r}, \xi_{2a}, \xi_{2r}$

The energy current of infinite big sphere Γ is calculated, we found the following energy current do not transfer energy:

1. *The energy current calculated from Poynting vector of the retarded potential,*

$$con(\xi_{1r}, \xi_{1r})_\Gamma = con(\xi_{2r}, \xi_{2r})_\Gamma = 0$$

2. *The energy current calculated from Poynting vector of the advanced potential*

$$con(\xi_{1a}, \xi_{1a})_\Gamma = con(\xi_{2a}, \xi_{2a})_\Gamma = 0$$

3. *The mutual energy of two retarded potentials,*

$$con(\xi_{1r}, \xi_{2r})_\Gamma = 0$$

4. *The mutual energy of two advanced potentials*

$$con(\xi_{1a}, \xi_{2a})_\Gamma = 0$$

In the above the symbol *con* means contributed energy to the outside of the surface.

These energy current can be seen as some kind of self-energy. Self energy do not exchange energy with other materials. It belong to the current itself.

Actually the in the first 4 principles, only the first two are required. We can prove the third and the fourth can be derived from the first two.

Proof:

Assume in the beginning there is only a current $\rho_1 = [J_1, K_1]$ in the empty space (their are no any background materials). The corresponding field is $\xi_1 = [E_1, H_1]$. We can calculate the Poynting vector as

$$Q_1 = (\xi_1, \xi_1) = \oint\!\!\!\oint_\Gamma (E_1 \times H_1^* + E_1^* \times H_1) \hat{n} d\Gamma$$

Assume now we put a second current to the space. The field in the space is $\xi = [E_2, H_2]$, the superimposed current is $\rho_2 = [J_2, K_2]$ is

$$\xi = \xi_1 + \xi_2$$

We have

$$Q = (\xi, \xi) = (\xi_1, \xi_1) + (\xi_1, \xi_2) + (\xi_2, \xi_1) + (\xi_2, \xi_2)$$

$$= Q_1 + Q_{12} + Q_{21} + Q_2$$

In the above we find Q_1 is still same as there is no current ρ_2 . Hence Q_1 is no thing to do with $\rho_2 = [J_2, K_2]$. $\rho_2 = [J_2, K_2]$ can be the whole environment materials, hence this part of energy do not change after, $\rho_2 = [J_2, K_2]$ appear. Assume $con(Q)$ is the contributed energy current of Q to the energy can be measured.

$$con(Q) = con(Q_1) + con(Q_2) + con(Q_{12} + Q_{21})$$

This part of energy current cannot be received, if we define the energy current as energy current can be received, which is $con(Q)$

$$con(Q_1) = 0$$

Even Q is not zero, but we only care the part of energy which can be received by other material. This part of energy is 0 for Q_1 .

In the same way, there is

$$con(Q_2) = 0$$

Since ξ is the retarded potential of $[J_1 + J_2, K_1 + K_2]$, the corresponding contribution of P-current should also vanish, i.e.,

$$con(Q) = 0$$

Assume ξ_1 and ξ_2 are retarded potential, the above discussion tell us that we have $con(Q) = 0$, $con(Q_1) = 0$ and $con(Q_2) = 0$, hence we have

$$con(Q_{12} + Q_{21}) = 0$$

$$\begin{aligned}
& con(Q_{12} + Q_{21}) \\
&= con((\xi_1, \xi_2) + (\xi_2, \xi_1)) \\
&= (\xi_1, \xi_2) + (\xi_1, \xi_2)^* \\
&= 2Re\{(\xi_1, \xi_2)\} \\
&= 2con(Q_{12})
\end{aligned}$$

or

$$con(Q_{12}) = 0$$

This tell us the contribution of the mutual energy current of the two retarded potential is 0. Similar this should be also the contribution of the mutual energy current of the two advanced potential is 0.

B. Only mutual energy between two object transfer energy

1. *The electromagnetic field energy is transferred through only a special kind of the mutual energy current, which is the current of one retarded potential and one advanced potential.*

We have add a condition for above principle, the currents are separated remotely. Since if they are too close their should be near fields, we do not clear how the near field transfer energy.

$$\begin{aligned}
(\xi_{1a}, \xi_{2r})_{\Gamma_1} &\neq 0 \\
(\xi_{1a}, \xi_{2r})_{\Gamma_2} &\neq 0 \\
(\xi_{1a}, \xi_{2r})_{\Gamma_3} &\neq 0
\end{aligned}$$

Here Γ_1, Γ_2 are the surfaces which enclosed the current ρ_1 and ρ_2 , Γ_3 is a plane between ρ_1 and ρ_2 . This principle is depended to the above principle. If P-current can not transfer energy, the only possibility of energy transfer is the mutual energy of a retarded potential and a advanced potential.

C. The mutual energy of a retarded potential and an advanced potential.

Assume ρ_1 is close to ρ_2 and close to the origin of a infinite big sphere, the corresponding field ξ_1 is retarded potential, ξ_2 is advanced potential. Here Γ is infinite big sphere surface.

$$Q_{12} = (\xi_{1a}, \xi_{2r})_{\Gamma} = 0$$

means that the mutual energy can only send energy between the transmitter to the receiver but not send any energy to the out space. We have proved this in Fourier domain. But this is also very easy to understand from time domain, the two retarded potential one is end to the future, one is send to the past, this two energy current can not be synchronized in the big sphere.

VIII. IMPORTANT RESULT TO PHYSICS

A. The advanced potential

The advanced potential cannot be vanish for receive antenna like a black body. We know this antenna has current J, K which does not vanish. We know black body do not send retarded potential out. Hence if there is no advanced potential, the field of J, K can only obtained from other transmit antennas. This kind field can not satisfy Maxwell equation. This means that we can not avoid the advanced potential. The author assume there exist the advanced potential to the any current $\rho = [J, K]$. The advanced potential is only depended to the current $\rho = [J, K]$ and it cannot be composed with other retarded potential. The advanced potential same as retarded potential always associated to the current ρ , even this ρ send also a retarded potential.

B. The P-current

If the energy current calculate through Poynting vector expresses a energy current, P-current of the advanced potential will bring a continuous energy to the current, even there is no any transmitter send energy out. This is very strange. Hence the author assumed that the P-current of the advanced potential do not carry any energy current. To make consistent, the author also assume the P-current of the retarded potential do not carry current energy.

This assumption, also solved the problem of so called 4-times difficulty and the difficulty by thin wire antenna. Hence in free space there is no any receiver, a current $\rho = [J, K]$ will produce a P-current for it's retarded potential and a P-current for it's advanced potential. This current belong to the current ρ itself, it do not exchange any energy with other current in the environment. The contribution to energy current of the P-current is 0.

C. Mutual energy

From above discussion we have known that the energy sent from transmit antenna to the receive antenna is the mutual energy current or M-current. If we calculate the field from the source, and calculate the energy current send out from the source using Poynting vector, i.e. P-current, which has no energy contribute to the receive antenna. In a few special situation the P-current can equal to the summation of all M-current between the transmitter and all the background environment, which can be seen as infinite small receivers. But in general for example in the case of thin wire antenna situation, the energy current can only be calculated through the mutual energy current

$$Q_{12} = (\xi_1, \xi_2) \quad (147)$$

where ξ_1 is retarded potential, and ξ_2 is the advanced potential.

D. Advanced potential sucks the energy from the transmitter

The energy received by receive antenna is equal to the energy sucked by the advanced potential of the receive antenna on the transmit antenna. The receive antenna play a important role in sending the energy out from the transmit antenna. This can lead very important physics consequence of the following. The sucked energy is transferred from the transmitter to the receiver is by the M-current or mutual energy current. The mutual current is a inner product of the advanced potential and the retarded potential.

People think advanced potential can not transfer energy, actually this is correct. Advanced wave itself really donot transfer energy. But actually the retarded potential itself also donot transfer energy. People thought retarded potential can transfer energy, this mistake obstruct us to understand the advanced potential. The energy transfer is by the inner

product of the advanced potential and a retarded potential. This inner product is just the mutual energy current of the retarded potential and the advanced potential. This mutual energy current will transfer the energy in the retarded meaning. That is the retarded potential and advanced potential together created the retarded energy current. Without the advanced potential, only with the retarded potential can not transfer any energy!

E. Reflection

Assume there is a mirror, the retarded potential of a source can be received by the mirror, in the mirror a current is caused by this retarded potential, the current will send an advanced potential to the transmitter and also send a retarded potential to surround materials. The advanced potential will cooperate with the retarded potential of the source to transfer energy through the mutual energy to the mirror. The retarded potential of the mirror will be sent to the materials surrounded the mirror. If the materials received the retarded potential and reacted it by sending an advanced potential back, the mutual energy current of the retarded potential and advanced potential can be transferred from the mirror to the materials.

F. The probability explanation of the quantum physics

We know that there is probability explanation for quantum physics. But why the light source send the photo randomly according to the probability? Einstein said God do not play dice. Many physicist do not agree the probability explanation of quantum physics, included Einstein and Schrödinger.

The authors believe the reason of the probability is because of the advanced potential and the mutual energy. A source send energy to the space can not be done by the source itself. It need the advanced potential of other receiver to suck the energy out from the source. If this sucking process is discrete, all the material surround the transmit antenna can be seen as many small receivers, which will randomly sucks the energy from the source or the transmitter. Hence the source can only send the photo randomly according to some probability.

This explanation can also avoid the concept of wave function collapsed. There is no the wave function collapsed after a receiver received a photon or a particle like electron. If the

advanced potential belong to a receiver, in this case perhaps it is a atom, in this situation the photon send out from the the transmitter, it is clear it will be received by which atom, that is the atom applied the advanced potential to the transmitter. It is not the wave function collapsed to the atoms (which can be seen as receivers). The photon is decided from the very beginning which atom it should reach. This like the atom (playing the role of the receiver) has stretched its hand to the transmitter and taken a photo back to the receiver. Hence the photon knows where to go. Hence the wave function collapsed can be avoid.

If electron wave transfer energy similar to photon by some kind of mutual energy and advanced potential and retarded potential, then, the electron will also similar to photon, the explanation for photon can be applied to electron. In this situation, The concept about Schrödinger wave function collapse can be avoid too.

If photo through their mutual energy cooperate the retarded potential and advanced potential to transfer energy, perhaps other particle do the same. In that case all probability explanation is because of the mutual energy.

This looks very strange if the distant between the receiver and the sender is very big for example many light year. The photo knows which atom is their destination. However if we accept the mutual energy and the advanced potential we have to accept also this very strange result.

G. In the space there is no any background material

In absorb theory[41–43], it said current send retarded P-current toward outside and received, and received P-current toward to itself. Hence the current have no gain and loss any energy.

In the case there is no any background material, if the current J, K situated on the center of space, it will send a retarded potential and a advanced potential continuously. The P-current of the retarded potential bring energy out from the transmitter and the P-current of advanced potential received energy current from out space. The current has no any pure energy loss, it can stay in space stably.

It is looks fine. But actually there is still problem, the advanced potential is send to past, and the retarded potential is sent to the future. If the current change is a short impulse. There should two impulse current send to the future and to the past. How can we measure

this two impulse?

We have successfully explain the energy transfer by a retarded potential and advanced potential. Only with retarded potential or only with advanced potential seems can only transfer energy mathematically and but not physically. It is better do not accept energy can be transferred with P-current.

Since we have assume there is no any material surround the transmitter, hence these two potentials both do not have any energy transferring to other material. Their retarded potential and advanced potential do not cause any energy toward the transmitter or toward the infinite big sphere.

H. Photon

The authors have to thought about what is photon. First if the receiver do not receive the retarded wave, is the receiver can send a advanced potential? That is same not possible. Hence the authors assume in the time the retarded potential reached the receiver that caused the current change in the receiver. However this current change send the the advanced potential, because it is advanced potential it has the ability to handshake with a past current in the transmitter, even that is a current belong to many many years ago. That means the photo is transferred between the two currents the sink and the source instantly. The word instantly is not enough to discernible how fast it is, because it use a minus time! It is better to create a word minus-time-instantly. So we can say that the photo runs from the the receiver to the transmitter minus-time-instantly. Here we do not speak about the photo runs from the transmitter to the receiver, instead it runs from the receiver to the transmitter. We have said that the receiver has the power to grab the energy from the transmitter. That is only our imagination. Actually, more accurate statement is the receiver direct send a photo to the transmitter through the advanced potential. The advanced potential is caused by the current in receiver, the current of receiver is caused by the retarded potential, but since the advanced potential runs so fast, minus-time-instantly, it still can combine with retarded potential to be come the mutual energy current. We have known that the mutual energy current is $(\xi_1, \xi_2)_T$, where ξ_1 is retarded potential, ξ_2 is advanced potential. This mutual energy current is just the photo's energy current.

We know that the 1-D retarded potential is

$$\phi_r(x, t) = \int_{-l}^l f(t - \frac{|x - x'|}{c}) dx' \quad (148)$$

advanced potential is

$$\phi_a(x, t) = \int_{-l}^l f(t + \frac{|x - x'|}{c}) dx' \quad (149)$$

for simplicity, assume $l \rightarrow 0$, $x' = 0$

$$\phi_r(x, t) = \delta(t - \frac{|x|}{c}) \quad (150)$$

$$\phi_a(x, t) = \delta(t + \frac{|x|}{c}) \quad (151)$$

assume $x > 0$,

$$\phi_r(x, t) = \delta(t - \frac{x}{c}) \quad (152)$$

$$\phi_a(x, t) = \delta(t + \frac{x}{c}) \quad (153)$$

or

$$\phi_r(x, t) = \delta(-\frac{1}{c}(x - ct)) \quad (154)$$

$$\phi_a(x, t) = \delta(\frac{1}{c}(x + ct)) \quad (155)$$

or, assume at $t = T$, the retarded potential reached the destination, it become, assume in time of $t = 0$ the retarded potential is sent out, the shape is following,

$$\phi_r(x, t = 0) = \delta(-\frac{1}{c}(x)) \quad (156)$$

Assume in time $t=T$, the retarded wave reached to the receiver,

$$\phi_r(x, t = T) = \delta(-\frac{1}{c}(x - cT)) \quad (157)$$

In this time the receiver send an advanced potential back to transmitter, the shape of the wave in this time should same as the reached retarded potential, i.e.,

$$\phi_a(x, t = 0) = \delta\left(-\frac{1}{c}(x - cT)\right) \quad (158)$$

Hence the advanced potential is

$$\phi_a(x, t) = \delta\left(-\frac{1}{c}(x - cT + ct)\right) \quad (159)$$

after a time $t = T$, the advanced potential travel to the following place,

$$\phi_a(x, t = T) = \delta\left(-\frac{1}{c}(x - cT + cT)\right) \quad (160)$$

$$= \delta\left(-\frac{1}{c}(x)\right) \quad (161)$$

We can see the advanced potential after a time $t = T$ it has the same shape with the the retarded potential.

Since the time t here is for the advanced potential, it is actually means a time in the past, this means that even the advanced potential is caused by the retarded potential, it still can synchronized with the retarded potential.

This means the light wave and light particle are different concept. The wave like property is because of the retarded potential. When the retarded potential from the transmitter reaches the receiver, there is still no energy particle send out, hence light is only waves. This is reason why light can have wave interference pattern. Later the current in the receiver begin to change, this change caused advanced potential, it can also produced their retarded potential which is scattering, here we do not like to talk about scattering. The receiver send the advanced potential out, this advanced potential reached a current in the transmitter of long time ago, and immediately grab the photon from transmitter to the receiver. In other words the photo runs from the receiver to transmitter minus-time-instantly. In this time a photon come from the transmitter to the receiver. Hence the photon process is always happens at the later time comparing the retarded potential.

Hence light wave and light particle are two concepts. Light wave is still go as wave in the space, but photon is only appear when the light wave reached the receiver. After this the current change on the receiver caused an advanced potential, the advanced potential

together with retarded potential produced a mutual energy current, this mutual energy current's energy is the photon.

Why energy is discrete? Advanced potential and retarded potential has to be synchronous. If the energy particle is too big, the advanced potential can become out of the phase with the retarded potential. Hence the energy particle cannot too big.

If the frequency of advanced potential has a little bit difference with the retarded potential or they have a very small speed difference, the two wave some where will cancel each other, some where will increase their magnitude, this can make the energy discrete.

Hence it is better the advanced potential has the very closed speed to the retarded potential. In this case the minus-time-instantly handshake between the receiver and the transmitter can happen. This is the photo. The photo is nothing else but the mutual energy current.

Energy particle (photon) is also possible caused by the change of the current in the receiver which must discrete. That is the energy particle only exist in the receive or transmit process. It is not exist in the space. In the space it is still the waves: the mutual energy of the retarded potential and advanced potential.

When we guess it is the Poynting vector transfer the energy, the photon is very difficult to understand, because the Poynting current is a diverge to the whole space, even for a light beam, the beam width is always increase. Why latter the retarded wave can collapse to one atom? Now we know the mutual energy transfer the energy, the beam of mutual energy of a transmitter to a receiver is not always diverge, it first diverge and then converge to the receiver or just a atom (if it is the receiver). Hence the mutual energy beam do not need to collapse to one point.

I. Spontaneous emission

From above we assume that the source is first send the retarded potential. However that is possible the receiver first send the advanced potential and that cause the source to have the spontaneous emission. Hence the principle of the mutual energy supported the absorb theory[41–43], which claim that the spontaneous emission is caused by the remote environment atoms in the future. The mutual energy solution offers a clear picture to this phenomena and tell us why this can happen.

J. Stimulated radiation

If the source produced a retarded potential ξ_1 , that will cause in the receiver has a current $\rho = [J, K]$, if this current also produced a retarded potential ξ_2 , the mutual energy of these two retarded potential will produce a mutual energy current. We have known that the mutual energy current cannot vanish in the big sphere for the two retarded potentials. This also mean that there will a mutual energy current send out to the free space. Corresponding to the quantum theory this phenomenon is stimulate radiation.

From the principle of mutual energy, looks this stimulate radiation is much different compare to the other above mentioned phenomenon which need a transmitter and a receiver and a advanced potential and a retarded potential.

Two retarded potential produced the mutual energy should be also belong to the P-current, which can not send energy current. It need advanced potential to receive their energy.

K. Speed of light

1. One possibility

From above discussion we have known that the energy current send from the transmitter to the receiver is the mutual energy current. The calculation of the mutual energy current needs the advanced potential of the received antenna.

The antenna system needs a advanced potential for receive antenna means that the speed of electromagnetic wave is depended on the speed of the advanced potential. The speed of advanced potential should be calculated from the time-space coordinates of the receive antenna.

Assume the the advanced potential has the speed of v_a and the retarded potential has the speed of v_r the speed of the mutual energy current should be some kind average of the two speeds, i.e.,

$$c = average(v_a, v_r)$$

Here c is light speed. The speed of mutual energy current offers us some method to guess what should be about the speed of light.

We have said that the advanced potential grab the energy from the transmitter, it perhaps also leads the retarded potential, hence the speed of retarded potential is the same to the advanced potential. This means the advanced potential is possible to be the ether of the retarded potential, hence the retarded potential has the same speed to the advanced potential, in this case we obtain,

$$c = \text{average}(v_a, v_a) = v_a$$

Hence the speed of light c is the same with that of the advanced potential. This will grantee the speed of light is a constant to the receiver.

We have also said the condition of the mutual energy current can transfer energy is that the retarded potential must synchronized with advanced potential. Hence the retarded potential must have same speed with the advanced potential. That is

$$v_r = v_a$$

Hence the retarded light speed must equal to the advanced potential. Hence the light speed is same with the receiver instead of the transmitter.

2. *Second possibility*

In the last sub-section we have said there is handshake between the receiver and the transmitter, this also ask that the retarded potential perhaps transferring in the same speed of the advanced potential.

However even if the retarded potential has different speed with advanced wave, the handshake process is only depended to the advanced potential. Hence we can allow that the retarded potential has the speed different with the advanced potential. But the light speed is depend on the advanced potential only, because the handshake process is controlled by the advanced potential.

Even the photo is instantly from transmitter go to the receiver, but because actually it is a handshake between a receiver current and a current of transmitter in perhaps many years ago, hence we feel that the photo needs a time to go from the transmitter to the receiver. The speed of this photon is the speed of advanced potential. This is way the light has the speed same as the advanced potential.

We can say that the speed of advanced potential is the speed of grab process or handshake speed which is also the photo's speed.

This way we have allowed the retarded potential has a different speed as the advanced potential. Assume the transmitter move toward the receiver in a speed v_t , assume the speed of retarded potential have a speed $c + v_t$ toward the receiver, assume the transmitter send a short impulse, when the retarded potential reached the receiver, the receiver send its advanced potential, which runs (with a reversed time) with a speed of c to the transmitter. In that time, the transmitter have move a distance from the original place, the short impulse has gone since the impulse has very short time. The handshake can not happen. No energy can transfer from transmitter to the receiver. This is not what we have seen.

This become very complicate if the retard potential has different speed with the advanced potential. It is better that we still assume the retarded potential has the same speed with the advanced potential.

Assume the retarded potential has different speed in the beginning, but when the advanced potential is returned from the receiver to transmitter, the retarded potential has to synchronized to the advanced potential, it has to be adjust to the same speed of the advanced potential.

3. third possibility

A difference of the mutual energy theory with special relativity is that the speed of light. According to the special relativity that the speed of light is same at any inertial frame. According to the mutual energy theorem, the speed of light is same to the receiver. It is the receiver send the advanced potential which is same as ether, the retarded potential must synchronized to the advanced potential, hence even in the beginning their speed can be any thing for example dependent to the speed of it's source, but after the receiver send back any advanced potential, the retarded potential must take the same speed as the advanced potential. If it's speed is different to the advanced potential, it can not synchronized and hence can not transfer any energy. We can assume the transmitter can send the retarded potential in a speed a little bit differ with light speed c and only the retarded potential with same speed of the corresponding advanced potential survived.

Hence there is the difference between the special relativity theorem and the point of view

of the mutual energy theorem. According to the authors point view we can build a system receiver the signal fast than the speed of light. For example, assume there is a transmitter which send the light signal. Assume I sit down at another place with a receiver, if i receive the signal, it's speed is c , that is clear. Assume I am sit down on big wheel, the wheel is rotated around me. In the rim of wheel there are detect which can receive the light signal. Hence this detector has different speed to me. The light received by this detector have also different speed. Even in my inertial frame, the light speed is different depending which detector receive it. Some will have speed larger than light speed c and some will have the speed smaller than the light speed c . Assume a detector received the signal speed fast than light, then it send me their signal, this do not caused any time. Hence this way we should get the signal a lit earlier than that received by myself. According special relativity all speed of light even it belong to different receiver. Hence detector at the rim of wheel cannot offer more fast to get the signal or less fast to get the the signal. They all get the same speed of signal.

According to mutual energy current(it is referred as M-current) theory the all light particle have the speed depending their receiver.

4. Forth possibility

Since the energy transferred by together of advanced potential and the retarded potential. Only when the two potential synchronized it can transferring energy. Hence the retarded potential must transfer in the same steed as the advanced potential. If all the receiver has the different speeds, the energy sucked in the source should has different speed corresponding to different receiver, hence the light speed is different corresponding the receiver. The advanced potential of receiver be come the ether of the light, any retarded potential with different speed with the advanced potential cannot synchronized and hence can not carry the energy. The speeds of photon in the transmitter is different depend their receiver.

Perhaps in the beginning the retarded potential has the same speed with the light speed in the transmitter's coordinates. But after the receiver's current changed, it send the advanced potential, the retarded potential immediately correct their speed.

L. Who is first, the retarded potential or the advanced potential

There is two possibility, first the transmitter has a current change, it cause a retarded potential, this retarded potential cause a current change in the remote receiver, the receive send a advanced potential back. In this case the retarded action is first, and the advanced reaction in the latter.

But in other situation the receiver has a current change it send advanced potential, this retarded potential reached the transmitter, the transmitter send a retarded potential back. Hence first have an advanced action, and then a retarded reaction,

The mutual energy formula do not forbid any of them, my feeling is the first has a little bit better for causality. That means the action is retarded is easy to understand. But from the absorb theory[41–43], the atom spontaneous decay is caused by the remote environment of future. This means the advanced action is also possible.

Any current should allowed to have a retarded potential and a advanced potential associated to it. Hence if there is any current change, it will produced two actions, one is retarded action direct to the future and an advanced potential directed to the past. That means the environment should be possible to receive advanced potential fir also.

M. Superluminal signal

Assume we have a wave guide, there is a transmitter in the left end and a receiver in the right end. The energy transferred from the transmitter to the receiver is because the mutual energy of the retarded potential and the advanced potential. If the voltage of the transmitter changed, the current in the transmitter is changed, the retarded potential is changed first, and this cause the the current of receiver change, this change in turn cause an advanced potential. The advanced potential cooperate with the retarded potential to become the mutual energy current, the mutual energy current send the energy from transmitter to the receiver in the retarded meaning. That means current in the transmitter take place first. The current in the receiver change later.

However if the current in the receiver is changed first, the advanced potential is sent from the receiver to the transmitter first. The transmitter current will change before the current change of in the receiver. This change in turn produced a retarded potential, the retarded

potential cooperated with the advanced potential to become the mutual energy current, sent from the receiver to the transmitter. The mutual energy current is sent from the transmitter to the receiver in the retarded meaning.

However the signal change is taken place at receiver first, but the change in the transmitter will take place at a time before the receiver change the current.

The current on the receiver can be changed by change their the resistance or impedance of the receiver.

There will be perhaps also a retarded signal have been return to the transmitter. We can find method absorb that return retarded wave. Hence this system only receive wave, if the resistance/impedance of receiver is changed, the transmitter must have a change before the change happens in the receiver. Hence the signal transmitted from the receiver with time $t = -T$.

In the space we can also by change the current in the receiver to send a signal to the transmitter. This signal will reach the transmitter in $-T$ time.

The experiment also can be done through two antenna located not too far away. Assume they the receive antenna is ideal, i.e., that it do not scatter energy. Assume there is no energy loss, that the energy sent from the transmitter antenna can be received by the receiver antenna hundred percent.

Now we transfer some wave for example microwave between them, every thing is fine suddenly we change the load resistance of the the receiver in the one end of the wave guide, that cause the current in the receiver is changed, this change will send an advanced potential to the transmitter, transmitter also adjust their current. The current change in the transmitter will happen before the resistance change in the receiver.

If we think the above is wrong, the receiver cannot send the advanced potential, we have known that this is ideal receiver antenna it can also not send the retarded potential out. Hence the transmitter cannot feel there is a load change in receive antenna, it will still send same energy to the receiver, but the receiver cannot absorb this energy. This is not possible. Hence if we assume the load in the receiver can affect the transmitter, it should send advanced potential to tell the transmitter its change! In this way we can transfer a sign with minus time. This is superluminal signal.

N. Causality

From the above discussion we found that even the advanced potential is runs backward or reverse with time, the photo, I mean the M-current, is still runs from past to the future.

However we can transfer the signal and energy from present to the past to a remote object, if we use the above superluminal method. If we make a circle wave guide the begin and end is in the same place, the signal can send to the same place but in the past. Hence the causality is possible to be violate.

O. Spin and polarization

First the polarization, we have been taught that the circle polarization is combined by two fields and between the two fields there is a 90 degree delay in phase. If photon is only related to one retarded potential it is very difficult to think about why in side the photo there have two fields, and the two fields have phase delay 90 degree. Hence we can only say that the photon has this spin property. This property is intrinsic. But this actually did not tell us any thing.

According to the authors M-current theory there are two field, ξ_1 and ξ_2 , one is retarded and one is advanced, but the two fields have been nearly synchronized in phase. If the transmitter send retarded potential $\xi_1 = [E_1, H_1]$, wen the receiver received the field ξ_1 , it send a advanced potential, $\xi_2 = [E_2, H_2]$, if this two potentials synchronized completely, it be come a line polarization. If the receiver/transmitter send a advanced wave have 90 degree delay, then it become a circle polarization.

If the receiver first send advanced potential ξ_2 , when the transmitter received this potential, it send retarded potential ξ_1 , ξ_1 will have 90 degree delay in phase. If the transmitter first send ξ_1 as retarded potential, when the receiver received it, and send ξ_2 as advanced potential, ξ_2 will have a 90 degree delay compare to ξ_1 .

We can assume the above transmitter and receiver are the two atoms, a emission atom and an absorb atom. Hence the photon or M-current has circle polarization is because the above described re-sending process of the potentials at two atoms. The existence of left and right polarization tell us, both receiver and transmitter can be the first one send the potentials.

Hence M-current theory can offer us a very good explanation about circle polarization of light energy. This explanation is much nature than tell us there is a spin for photon.

P. Why is this special atom react to the retarded potential

A transmitter send a retarded potential to the environment, there is infinite atoms received this potential, why that special atom send back a advanced potential not others? We can assume the atom in the receiver can be awake only in a very short time to receive energy from the retarded potential, for example 10^{-10} second. Other time it just sleep do not react to the retarded potential. The time in which a atom can receive energy become a random variable. In special time only a few atom can receive energy. This short time window is also the reason why the photo is also a short time energy particle.

Transferring through the mutual energy with two potential retarded and advanced can make the the energy focused to a special atom, The above time window make the energy also focused to a short time. This too functions create a photon particle with a local place in a short time. That amount energy can be seen as photon. Hence photon is not a particle in space, it is only a particle in the time it is created and it is received. In other time it is retarded potential and advanced potential.

Q. Action with a remote object with 0 time

We have known the current change in the transmitter cause a retarded potential, the retarded potential applied an action to the receiver. The receiver give a reaction back. This reaction perhaps is the photon, the receiver sent to the transmitter minus-time-instantly. What is the time the reaction applied to the transmitter? Since the the retarded potential transfer from transmitter need some time. The advanced potential apply the reaction to transmitter need the time which is negative. Together the time is just zero.

This means for transmitter it can fell the object in any remote distance with a 0 time. This means the receiver can apply a reaction to the transmitter immediately with no time. The transmitter can feel a force from the receiver, the reaction is immediately.

If the gravitational field has the similar functions like electromagnetic field, also have the advanced potential, the force reaction between stars are also with 0 time. It is seems it is

correct that we normally believe there is no need of time to apply a gravitational force from a star to another. If it is true that means the gravitation should also have the advanced potential for the gravitational waves. And the force reaction can be with 0 time between far away stars.

The concept of mutual energy and mutual energy theorem, the principle of mutual energy support many results of absorb theory[41–43], but give more details what the mutual energy can be calculated. How the energy is transferred from atoms. And also give the picture why only retarded potential alone and advanced potential alone do not work, but their combination works.

The concept the energy is transferred through the mutual energy of a retarded potential of transmitter and advanced potential of the receiver allow us to obtain the action and reaction only take 0 time means that the electromagnetic force can be done with 0 time. If the gravitation force has same principle with electromagnetic field, it should also can action with remote object with 0 times.

R. Mass

We have known there is two kind of mass m_a , the mass because of acceleration and the mass of the gravitation m_g , experiment shows this two mass are equivalent, that means

$$m_a \propto m_g \tag{162}$$

From above discussion we have known that the object can apply a electromagnetic action to the remote environment, and this action with remote environment do not need time. This means it can linked with the environment with this fast action. The environment can apply a reaction to the object with 0 time. Hence if we try to accelerate a object, we will fell a force to us. This force is the reaction of far away environment.

If gravitation is also cased by action and reaction between two objects. The action and the reaction transferred perhaps same with a electromagnetic field, mutual energy of the retarded potential and advanced potential. In this case the two mass can be equivalent to each other, i.e., there is Eq.(162).

S. Quantum entanglement

When we talk about quantum entanglement, we often said that there is no any method to link the two electrons, to let one tell the other what is their state of spin. Know at least we know there is superluminal signal can connect the two electrons. Even we do not know exactly what happens for entanglement but their signal perhaps is the 0 time signal like the force reaction or superluminal signal of electromagnetic signal.

IX. NEW EXPLANATION OF THE WAVE FUNCTION FOR THE QUANTUM PHYSICS

A. Electron

Electron should also had its advanced potential. Electron should also some kind of mutual energy theorem for electron, that means there perhaps is another kind of more accurate wave function than current Schrödinger or Dirac equations, so that the electron also have the retarded potential and the advanced potential, this make the electron transmitter can communicate with another electron receiver. The electron transmitter an atom. The electron receiver is another atom. The electron itself is wave like light wave. the electron's mutual energy beam should similar to the mutual energy of the light beam, it is very narrow in both transmitter point and at the receiver point. The electron M-current beam should be very wide in the place between the transmitter and the receiver.

Time window of the electron receiver or transmitter can react should be very narrow. Hence there is only one receiver to react the retarded potential send out from the transmitter. This particular transmitter atom and the particular receiver atom have a handshake or marriage. All energy of a electron is send from this particular transmitter atom to another particular receiver atom.

B. Wave function

In the quantum physics, assume ψ is the wave function, then $|\psi|^2$ is explained as probability. However the authors thought that is because the lack of the knowledge of M-current for light. If 90 years ago Schrödinger and Dirac knew the above theory about light they will

build their quantum theory looks like light. Here the author means the theory which explain light as M-current. The M-current is composed of retarded potential and an advanced potential.

After we have the new understand above about light, we know that, the situation of quantum physics should be similar to electromagnetic wave or light wave. If we accept the advanced potential in electromagnetic field and light, if we accept the light is just the mutual energy current of the two potentials, one is retarded potential, another is advanced potential, we can immediately thought in the quantum physics perhaps there is also the advanced potential. Assume ψ_1, ψ_2 are two potentials, we can define the the M-current (mutual energy current) of quantum physics as

$$Q_{12} = (\psi_1, \psi_2) = \psi_1 \psi_2^*$$

It is possible that ψ_1, ψ_2 are vectors like in the electromagnetic field situation, in that situation (ψ_1, ψ_2) will define the mutual energy current. In quantum physics ψ_1, ψ_2 are scales (the scale is possible only a simplified version of vector field, just light in optics we can use scale value to describe the electromagnetic field, which actually is a vector field). In case ψ_1, ψ_2 are scales, $(\psi_1, \psi_2) = \psi_1 \psi_2^*$.

Assume in quantum physics, ψ_1 is retarded potential which send out from a transmitter atom, ψ_2 is advanced potential which is send out from a receiver atom.

When the electron is in side the orbit of the atom, the advanced potential can be completely synchronized with the retarded potential. In this case $\psi_2 = \psi_1$, Assume the field of electron can be superposed

$$\psi = \psi_1 + \psi_2$$

ψ is the electron field inside the orbit. Hence

$$\psi_1 = \psi_2 = \frac{1}{2}\psi$$

$$(\psi, \psi) = (\psi_1 + \psi_2)(\psi_1 + \psi_2)^*$$

$$= \psi_1 \psi_1^* + \psi_1 \psi_2^* + \psi_2 \psi_1^* + \psi_2 \psi_2^*$$

$\psi_1\psi_1^*$ is the retarded potential's self-energy current.

$\psi_2\psi_2^*$ is the advanced potential's self-energy current.

$\psi_1\psi_2^* + \psi_2\psi_1^*$ is the mutual energy current.

In this situation since $\psi_1 = \psi_2$, the calculation only with retarded potential, and assume that is ψ will obtain the same result. This is why if we do not introduce the concept of advanced potential and the M-current, quantum physics is still obtain corrected results in the situation of electron inside orbit.

C. Election in the free space

The self-energy current $\psi_1\psi_1^*$, $\psi_2\psi_2^*$ have no contribution to the transmitter atom and the receiver atom, this is similar to the situation of light. $\psi_1\psi_1^*$ is a beam diverged from the transmitter, when it reach the receiver atom, since the the section area of the receiver atom is too small, the receiver atom received energy from the $\psi_1\psi_1^*$ can be omitted. $\psi_2\psi_2^*$ is diverged from receiver, when it reached to the transmitter atom, since the section area of the transmitter is too small, this part of transferred energy can be omitted. In this situation only the mutual energy is important, which is

$$\psi_1\psi_2^* + \psi_2\psi_1^* = 2Re\{\psi_1\psi_2^*\}$$

The mutual energy current similar to the situation of light the beam is that the electron beam first diverged from the transmitter and then converged to the receiver. Here the transmitter and the receiver are two atoms which can send or absorb the electrons. Here since the beam of M-current can focus to a small point, it does not need the concept of wave function collapse.

The wave function collapse is because we do not know there is also the advanced potential. So We calculate $\psi_1\psi_1^*$ which is a diverged beam. At the place of the wave is received, the beam of the energy current $\psi_1\psi_1^*$ become very widely spread out. When we $\psi_1\psi_1^*$ is the result of quantum physics, we have to face the wave function suddenly collapsed to a point. After we explain electron as M-current, the property of first diverge and converge can thoroughly avoid the wave function collapse.

The probability explanation for the wave function is only because of we only calculated the retarded potential, $\psi_1\psi_1^*$.

The author don't clear why this particular transmitter atom married to another particular receiver atom. We have said it is because perhaps because just in the time the retarded potential reached the receiver atom, their time window matched together. But this is only one possibility, that is also possible the transmitter send retarded potential includes a special cryptograph code, which can be understand only one receiver atom. It is also has some positive feedback between the transmitter and the receiver that makes the connection of one pairs of atoms become strong than others. Finally they become marry together. The electron is send out from the transmitter atom to the receiver atom.

D. Spin

In the traditional quantum physics, there is only one wave function, when we measured some thing rotated, it is difficult to give an explanation, hence we call it spin. However in the authors' new quantum explanation, there are two wave functions, one ψ_1 is retarded and the other ψ_2 is advanced. The two waves are nearly synchronized. But there is allow they have small phase difference. The spin is also similar to the situation of light. If we assume ψ_2 has 90 degree phase difference compare to ψ_1 , there is a circle polarization. Here we assume ψ_1 and ψ_2 are transverse field. This circle polarization is the so called spin.

In the explanation of the mutual energy current, the spin just two waves have a 90 degree phase difference. This phase difference is caused by the receiver atom or the transmitter atom there is a reaction delay to their wave re-sending process.

E. The Schrödinger equation considered the advanced potential

The original Schrödinger equation which is corresponding to the retarded potential

$$ih\partial\psi(x,t) = [-\frac{h^2}{2\mu}\nabla + V(x,t)]\psi(x,t)$$

Corresponding to the advanced potential, there is

$$-ih\partial\psi(x,t) = [-\frac{h^2}{2\mu}\nabla + V(x,t)]\psi(x,t)$$

The above is only one example to create the advanced potential, we also can created the advanced potential using Klein-Gordon equation or Dirac equation, or any other equation

still not found, but that is beyond the discussion of here.

F. Summary

For a free electron, we should calculate M-current which $\psi_1\psi_2^*$. M-current is a beam first diverge and then converge, for this kind wave beam, the concept of wave function collapse can be avoid.

When the electron inside orbit, the two potential can synchronized completely, and hence the retarded potential and advanced potential is equal to each other. In this situation, the calculation of $\psi_1\psi_1^*$, the P-current (we can referred it as P-current similar to the light situation) is same as M-current $\psi_1\psi_2^*$. Even there still exists the advanced potential, but the result is same when we only use retarded potential to calculate, $\psi_1\psi_1^* = \psi_1\psi_2^*$. It is same to the wave guide, in the orbit, the energy transferred half by P-current $\psi_1\psi_1^* + \psi_2\psi_2^*$ and half by M-current $\psi_1\psi_2^* + \psi_2\psi_1^*$

In the free space the contribution of P-current can be omitted completely. Only M-current is left. Hence electron is also M-current, which is composed of two waves retarded potential and advanced potential.

Even the above new explanation has not change the calculation of the quantum field. However because it abandons the concept of the wave function collapse, the probability, thing become easy to understand. Electron in the free space is nothing else, it is just M-current. The M-current is composed of two waves one is retarded the other is advanced. The two wave are nearly synchronized. There is 90 degree phase difference which can be seen as the behind scene of spin.

In the authors new explanation the square absolute value of wave function $\psi_1\psi_1^*$ is the P-current which is only a approximation to the mutual energy $\psi_1\psi_2^*$. Since ψ_2^* is difficult to obtain, there is hundreds and thousands ψ_2 in the environment corresponding to each atom which can receive the electron, we still only calculate ψ_1 . In this situation we have to use the probability to explain the result $\psi_1\psi_2^*$.

X. THE PROBLEM IS STILL NOT SOLVED

Since this new theory, there are lot of thing need to do. I list some of that as following,

A. Why action and reaction is same?

Newton has the third law, that the action force is same to the reaction force.

We can assume the force is caused by a action try to accelerate the object, when a object accelerate it send a retarded potential to the remote environment, the remote environment send an advanced potential back to the object. this become the reaction, the reaction happens with 0 time because the advanced potential travel minus-time-instantly. But why this reaction is same to the action?

B. Planck constant?

Why has this Planck constant? Is this constant can be explained with the concept of the mutual energy?

C. In the loss media the mutual energy theorem is not established, what should be the mutual energy theorem in that special situation?

If the advanced potential has different transfer character, for example the media for the advanced potential is not the same as with retarded potential, how energy will transfer? For example if we let the advanced potential go a way different with the retarded potential and then put it back to the same way as before, is it possible to transfer energy?

If the energy is transferred by M-current when the retarded potential and the advanced potential is separated, is the energy can be transferred? If it can still transfer the energy. Dose this mutual energy current still has the meaning?

We know the fields can still transfer, hence in this situation should be still possible to transfer the energy, then what is wrong with the mutual energy current?

If the advanced potential can be separated with retarded potential and the energy still can be transferred, that means the mutual energy is also not the one transferring energy, the energy is transferred only with the retarded potential and the advanced potential.

This problem I still did clear.

D. What about polarization to the advanced wave?

Is the polarization property belong to retarded potential or advanced potential or both? If it is the property of the retarded potential, is the advanced potential has the same property of polarization?

E. Optics to advanced wave

For example what is the Huygens principle to the advanced potential?

F. The theory for the whole microwave network

If the theory about mutual energy and advanced potential is correct, the whole electric network theory need to be updated. For example only for coaxial cable, originally we can tell students that the transferred energy can be calculated using Poynting vector, which can be calculated with only retarded potential. Now we must tell that is wrong, actually the energy is transferred by the retarded wave and advanced wave together, it is the mutual energy transfer the energy. But in this situation the result calculated by the mutual energy is same as the results calculated the energy current with Poynting vector.

XI. CONCLUSION

We have shown that the traditional way to explain the system with a transmit antenna and a receive antenna using the reciprocity theorem is inadequate, which is only correct to the system with two transmit antennas. Instead it is need a new explanation. In the new explanation the mutual energy and advanced potential is involved.

We have shown that the energy current related to the Poynting vector does not transfer any energy. It is the mutual energy current that is responsible to transfer the energy from the transmitter to the receiver.

We have also shown that the advanced potential can suck energy from the transmitter. This sucked energy is just equal to the energy received by the receiver.

The process of advanced potential sucking the energy should be also discrete. This discrete energy is just the photon.

The background material can be seen as countless receivers, these receivers will randomly apply their advanced potential to the transmitter, hence the transmitter randomly send their energy or photon out. This is the reason that the quantum mechanics result has to be explained with a probability. We also discussion the reason of speed of the light. We also discuss how to transfer a superluminal signal.

Part II

Photon model in Time domain

... It will be added later.

Part III

The Principle of Mutual energy

XII. THE PROBLEM OF THE SUPERIMPOSITION OF THE FIELD

A. History of field theory

In the time Newton, there is action and reaction. action and the reaction is equal. This is Newton's third law. This is no any problem, for newton because that time the action can be seen with infinite fast speed. But today we know the action of electromagnetic field have the speed c it is not infinity any more. Hence the action at distance change to two viewpoint. (1) the emitter send action to the field, the field propagate to the absorber, then the absorber receive the action from the absorber. (2) the action is still same as newton said, action is still equal's the reaction. The action is only take place between the charges.

The first view of point (1) is very successful in the history of classical electromagnetic field theory. Later become the Maxwell equation and theory. This we all clear. The second view of point (2) is not well know, it is try to solve the problem of the first one. We all know the Maxwell electromagnetic field theory has some problem which lead the self action infinity.

We can not calculate a energy of a charge in a point. That create a infinity. Quantum physics has to do the process of re-normalization to eliminate the infinity. In quantum physics Maxwell theory tell us the wave have send to all direction through Poynting vector, however we always receive a energy package as photon, hence we have to speak about wave collapse. However no one tell us what equation satisfy by this collapse process. Schwarzschild first offers the results for the second view of point. His theory has been referred as direct interaction theory. Tetrode and Fokker further developed this theory and offers the same principle but suitable to relative theory. In this direct interaction theory, the action is still only between the two charges. There is no field exist without any one of the charge.

The second view of point continually developed by Dirac and later Wheeler and Feynman[42] as absorber theory and adjunct field theory. The adjunct field that means it is only adjunct to the action of the two particle, it is not a independent field. Hence the real calculation still need to use Maxwell equation. Wheeler and Feynman tell us

“(1) There is no such concept as “the” field, an independent entity with degrees of freedom of its own.”

“(2) There is no action of an elementary charge upon itself and consequently no problem of an infinity in the energy of the electromagnetic field.”

“(3) The symmetry between past and future in the prescription for the fields is not a mere logical possibility, as in the usual theory, but postulation of requirement”

When I read the above, my heart spring out. When I now still blind try to solve the self-action problem, our forefathers has loon clear tell us the results. Wheeler and Feynman claim they can derive the Maxwell equation from this adjunct field theory. Actually they derive the Maxwell equation for a charge.

Wheeler and Feynman offers the the principle what should to solve the problem but not really gives a new electromagnetic theory which can replace the Maxwell equation or correct the Maxwell equations. This will be our task.

B. Poynting theorem is equivalent to Maxwell equation in principle

We can derive the Poynting theorem from Maxwell equation. We do not know how to derive Maxwell equation from Poynting theorem. However we can derive all reciprocity theorem (also the mutual energy theorem) from Poynting theorem[XXXX]. We also know

the Green function theory can be derived from reciprocity theorem. Maxwell equation can be solved through green functions. From all solutions of Maxwell equations, the Maxwell equation can be derived by induction. Our proof is not very stringent. But physics is not mathematics, this kind of proof is good enough.

In the following if we cannot easily find something wrong in Maxwell equation, we can try to stack the Poynting theorem. If Poynting theorem can be corrected, in the same way we actually correct the Maxwell's theory.

C. Is field can be superimposed?

There are two versions of superimposition of the field, for example there are N charges. The first one is the all contribution of the charges

$$\vec{E}(x) = \sum_{j=1}^N \vec{E}(x_j, x) \quad (163)$$

This is our traditional way to define the field. Maxwell follows this way. However this definition is only suitable to the situation the field is calculated not on the position of charges.

$$\vec{E}(x_i) = \sum_{j=1, j \neq i}^N \vec{E}(x_j, x_i) + \vec{E}(x_i, x_i) \quad (164)$$

but

$$\lim \vec{E}(x_i, x_i) = \infty \quad (165)$$

Hence we change to the following definition

$$\vec{E}(x) = \begin{cases} \sum_{j=1}^N \vec{E}(x_j, x) & x \notin I \\ \sum_{j=1, j \neq i}^N \vec{E}_j & x \in I \end{cases} \quad (166)$$

$I = 1, \dots, i, \dots$, it is the sets of position of the charges. The above section definition is also not satisfactory. Many will ask is this correct that the field is extended to any position without a charge?

According to the adjunct field theory of Wheeler and Feynman, that the field can only be defined on the position of charges, because the force is only defined on the position of the charges.

$$\vec{F}(x_i) = \sum_{j=1, j \neq i}^N \vec{F}(x_j, x_i) \quad x \in I \quad (167)$$

Hence

$$\begin{aligned} \vec{E}(x_i) &= \frac{1}{q_i} \vec{F}(x_i) \quad x \in I \\ &= \sum_{j=1, j \neq i}^N \frac{1}{q_i} \vec{F}(x_j, x_i) \quad x \in I \\ &= \sum_{j=1, j \neq i}^N \vec{E}(x_j, x_i) \quad x \in I \end{aligned} \quad (168)$$

Since the force only defined at the position charge, the field should also only defined at the position of the charge.

We believe the theory of Wheeler and Feynman is correct, because if in the empty space there are only two charges, if the action of the first can produce a field and action with the field, send the energy to the field, the most energy will go outside the system of this two charges. The second charge can only get a very small part of energy from the first charge. The energy is not conserved for this two particle system.

But if the field cannot be defined on the space other than the position of charges, it become not very useful. Here we at least has 3 different field definitions. Hence field is a very confused concepts.

D. Energy or Power

From last subsection we know the field is very confused concept, if we cannot correctly define the field, let us see if we still can apply the concept of energy or power.

If the charge move and has the speed \vec{v}_i , we can define the Power which is

$$P(x_i) = \vec{F}(x_i) \cdot \vec{v}_i \quad (169)$$

We believe this still correct. Hence the power of the whole system with N charges will be,

$$P = \sum_{i=1}^N P_i = \sum_{i=1}^N q_i \vec{E}(x_i) \cdot \vec{v}_i = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}(x_j, x_i) \cdot (q_i \vec{v}_i) \quad (170)$$

We can write

$$\vec{J}_i = q_i \vec{v}_i \quad (171)$$

where \vec{J}_i is the current of charge q_i . Hence we have the Power of the whole system are

$$P = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}(x_j, x_i) \cdot \vec{J}_i \quad (172)$$

We find when we calculate Power, we have used the following summation.

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \quad (173)$$

The above power calculation is no disputed, why we do not started from this to redefine what is field? In the above we have offers 3 version of field definition no one is satisfied all situations.

For the whole system

$$P = Const \quad (174)$$

The above is energy conserved law, the whole system energy

Hence we define field as following,

$$\vec{E}(x) = [\vec{E}(x_j, x), \dots E(x_j, x) \dots] \quad (175)$$

or

$$\vec{E}(x) = [\dots \vec{E}_j \dots] \quad (176)$$

In the above definition, we can define a field to any place where is not at the position of the charge. However we give up the superimposition principle. We do not clear how to “add” the fields of many particles. What we know is the Power of the whole system still can be given.

E. The Poynting theorem of N charges

According to the traditional electromagnetic field theory, The Poynting theorem is give as following,

$$\begin{aligned}
& -\nabla \cdot (\vec{E} \times \vec{H}) \\
& = \vec{E} \cdot \vec{J} \\
& (\vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B})
\end{aligned} \tag{177}$$

According to the Traditional definition, Here we still not apply our new field definition.

$$\vec{E} = \vec{E}_1 + \cdots + \vec{E}_i + \cdots + \vec{E}_N \tag{178}$$

$$\vec{H} = \vec{H}_1 + \cdots + \vec{H}_i + \cdots + \vec{H}_N \tag{179}$$

We have

$$\begin{aligned}
& -\nabla \cdot (\sum_j \sum_i \vec{E}_i \times \vec{H}_j) \\
& = \sum_j \sum_i (\vec{E}_i \cdot \vec{J}_j) \\
& + \sum_j \sum_i (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)
\end{aligned} \tag{180}$$

In last subsection we see the summation

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \tag{181}$$

But it is not used in N system Poynting theorem. We use this new summation to replace the original summation, we obtain,

$$\begin{aligned}
& -\nabla \cdot (\sum_j \sum_{i \neq j} \vec{E}_i \times \vec{H}_j) \\
& = \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j)
\end{aligned}$$

$$+ \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \quad (182)$$

or

$$\begin{aligned} & - \oint_{\Gamma} (\sum_j \sum_{i \neq j} \vec{E}_i \times \vec{H}_j) \cdot \hat{n} d\Gamma \\ & = \iiint_V (\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j)) dV \\ & \iiint_V (\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)) dV \end{aligned} \quad (183)$$

It is the rest items of Poynting theorem if all self items as following

$$\begin{aligned} & - \oint_{\Gamma} (\sum_i \vec{E}_i \times \vec{H}_i) \cdot \hat{n} d\Gamma \\ & = \iiint_V (\sum_i (\vec{E}_i \cdot \vec{J}_i)) dV \\ & \iiint_V (\sum_j (\vec{E}_i \cdot \partial \vec{D}_i + \vec{H}_i \cdot \partial \vec{B}_i)) dV \end{aligned} \quad (184)$$

are taken away. This is the the mutual energy theorems[XXXXXXXXXX]. This formula is correct in two ways. (1), if Maxwell equation is correct this formula is also correct, it is easy to prove this. Because we take away all self items all satisfy Poynting theorem for a single charge. From the Poynting theorem of N charges take away all corresponding Poynting theorem for single charge this guarantees the rest part still satisfies Maxwell equations, since Poynting theorem can be derived from Maxwell equations. (2) The second way to show this formula is correct is it satisfies also the “direct interaction” theory. This formula actually offers a correct definition of the adjunct field of the Wheeler and Feynman. In the following we show this formula satisfy the direct interaction theory. In this formula the left side is

$$P = \iiint_V (\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j)) dV \quad (185)$$

which clear is the whole power of the system with all charges same as last section. The first term of right side is

$$- \oint_{\Gamma} \left(\sum_j \sum_{i \neq j} \vec{E}_i \times \vec{H}_j \right) \cdot \hat{n} d\Gamma \quad (186)$$

is the power send to outside space, it is the energy current send to outside of system. If there is only N charge in a empty space, there should no energy current go outside. We have know from the mutual energy theorem if photon's field either retarded field for the emitter or advanced field from absorber, the mutual energy current vanishes. Hence this term is 0.

The following is

$$\iiint_V \left(\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \right) dV \quad (187)$$

is the system energy in the space. If started from some time there are no action or reaction to a end time there are also no action and reaction. The integral of this energy is vanishes, i.e.,

$$\int_{t=-\infty}^{\infty} \iiint_V \left(\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \right) dV dt = 0 \quad (188)$$

Hence we have the last term

$$\int_{t=-\infty}^{\infty} \iiint_V \left(\sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j) \right) dV dt = 0 \quad (189)$$

This term also vanishes. The above formula tell us system all energy is conserved. Hence this corrected formula. It is much meaning full compare to the the original formula of Poynting theorem. This formula actually is the mutual energy theorem we also call it mutual energy principle (in the past we call it principle is because we believe it also can be established on quantum physics for example the theory of electrons instead photons. In that time we still not decided to replace Maxwell equation with this mutual energy theorem).

Compare to Poynting theorem, it have

$$\iiint_V (\vec{E}_i \cdot \vec{J}_i) dV = \infty \quad (190)$$

If the charge is a point.

$$-\oint\oint_{\Gamma}(\vec{E}_i \times \vec{H}_i) \cdot \hat{n}d\Gamma \neq 0 \quad (191)$$

The system always some energy go to outside even there is empty without other charges, which we do not understand. It is very strange. It is clear Poynting theorem conflicts with direct interaction principle. But the mutual energy theorem doesn't.

F. Looking for new principle to replace the Maxwell equation

In the above we have prove the Poynting theorem is problem as physic theorem. This wrong also lead the Maxwell equations fail. We have said the Poynting theorem is nearly equivalent to Maxwell equations.

We have shown that mutual energy theorem is correct and satisfies energy conservation and direct interaction principles. Why we do not started from mutual energy theorem and build the whole electromagnetic field theory?

G. Upgraded the mutual energy theorem as mutual energy principle

We have tried a lot times to prove the self energy items vanishes in the Maxwell theory, it seems successful with a return process, that means the field is end to infinity some reason it can return back. The return wave satisfy time-reversed Maxwell equations.

However there is always something doesn't very self consistent. Its time for us to solve this problem thoroughly. We believe the problem is at the Maxwell equations. Hence we will replace the the Maxwell equations.

Now we speak about the mutual energy principle instead of mutual energy theorem. This is because we would like to replace Maxwell equations by the mutual energy theorem. After the replace the mutual energy theorem upgraded to as the mutual energy principle.

H. What is the conditions for a principle

Wheeler and Feynman offers the 3 conditions to started the electromagnetic theory which are,

“(1) Well defined. (2) economical in postulates and (3) in agreement with experience.”

We know that Maxwell equation and the mutual energy theorem are all well defined. In the above sections we have clear shows that the mutual energy theorem is agree with energy conservation and direct interaction principle. Poynting theorem and hence Maxwell equation doesn't satisfy (3).

Now we look the condition (2), economical in postulates.

In the above when we speak the Maxwell equation has problem, we mean it is problem for a system with N charges. In case there is only two charges, one is emitter and another is absorber, Maxwell equation still possible to be correct in the following way.

If we assume take mutual energy theorem as principle, we need to solve the equation of the mutual theorem equation which for only two charges are following,

$$\begin{aligned}
& -\nabla \cdot (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \\
& = \vec{E}_2 \cdot \vec{J}_1 + \vec{E}_1 \cdot \vec{J}_2 \\
& + \vec{E}_1 \cdot \partial \vec{D}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{H}_2 \cdot \partial \vec{B}_1
\end{aligned} \tag{192}$$

This can rewritten as

$$\begin{aligned}
& -(\nabla \times \vec{E}_1 \cdot \vec{H}_2 - \vec{E}_1 \cdot \nabla \times \vec{H}_2 + \nabla \times \vec{E}_2 \cdot \vec{H}_1 - \vec{E}_2 \cdot \nabla \times \vec{H}_1) \\
& = \vec{E}_2 \cdot \vec{J}_1 + \vec{E}_1 \cdot \vec{J}_2 \\
& + \vec{E}_1 \cdot \partial \vec{D}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{H}_2 \cdot \partial \vec{B}_1
\end{aligned} \tag{193}$$

or

$$\begin{aligned}
& \vec{E}_1 \cdot (\nabla \times \vec{H}_2 - \partial \vec{D}_2 - \vec{J}_2) + (-\nabla \times \vec{E}_2 - \partial \vec{B}_2) \cdot \vec{H}_1 \\
& + \vec{E}_2 \cdot (\nabla \times \vec{H}_1 - \partial \vec{D}_1 - \vec{J}_1) + (-\nabla \times \vec{E}_1 - \partial \vec{B}_1) \cdot \vec{H}_2 \\
& = 0
\end{aligned} \tag{194}$$

from this we derive that, if

$$\nabla \times \vec{H}_2 - \partial \vec{D}_2 - \vec{J}_2 = 0 \tag{195}$$

$$-\nabla \times \vec{E}_2 - \partial \vec{B}_2 = 0 \quad (196)$$

$$\nabla \times \vec{H}_1 - \partial \vec{D}_1 - \vec{J}_1 = 0 \quad (197)$$

$$-\nabla \times \vec{E}_1 - \partial \vec{B}_1 = 0 \quad (198)$$

The mutual energy theorem is satisfied. The nonzero solution of the mutual energy theorem ask the field $[\vec{E}_1, \vec{H}_1]$ and $[\vec{E}_2, \vec{H}_2]$ nonzero in the same time. The above formula are Maxwell equation for singular charges,

$$\begin{cases} \nabla \times \vec{H}_1 = \vec{J}_1 + \partial \vec{D}_1 \\ \nabla \times \vec{E}_1 = \partial \vec{B}_1 \end{cases} \quad (199)$$

and

$$\begin{cases} \nabla \times \vec{H}_2 = \vec{J}_2 + \partial \vec{D}_2 \\ \nabla \times \vec{E}_2 = \partial \vec{B}_2 \end{cases} \quad (200)$$

Hence if we choose the mutual energy theorem as the principle, the Maxwell equation still can be obtained as sufficient conditions of the mutual energy theorem.

In other hand if we choose Maxwell equation as principle, the above 4 formula is still not enough, we have to add a special condition ask the two fields $[\vec{E}_1, \vec{H}_1]$ and $[\vec{E}_2, \vec{H}_2]$ nonzero in the same time. This even not enough, we still have to explain any place in the space if one of field vanish $[\vec{E}_1, \vec{H}_1]$, the other field loss its meaning $[\vec{E}_2, \vec{H}_2]$.

For example if the emitter and absorber is at two sides of the metal plate. In the metal plate there is a hole. the fields of the emitter and the absorber become cone beam fields. The two cone beams has a overlap region. The field can only nonzero in this overlap region simultaneously. In other place there is only one field ether $[\vec{E}_1, \vec{H}_1]$ and $[\vec{E}_2, \vec{H}_2]$ is nonzero. Hence in the region which is not the overlap region, the field is not meaning any more even there still one field not vanish according to the Maxwell equation.

If we use the mutual energy theorem as principle, this is clear, we only care the overlap place where the mutual energy is meaningful. Hence if we use mutual energy theorem as principle we need only one formula. If we use Maxwell equations we need 4 formula and plus a very strangle conditions. This means if we use the mutual energy theorem as principle, it is much more economy than use of the Maxwell equations.

I. The necessary condition of the Mutual energy theorem

We just have said that the Maxwell equations plus the two fields are nonzero simultaneously are the sufficient conditions. It is not the necessary condition. There is the possibility that the Maxwell equation doesn't satisfy but the mutual energy theorem still satisfied. If we found this kind solution in the real world. The Maxwell equations will thoroughly loss its power as a principle.

J. Maxwell equations in Macrocosm

We know that we never can prove Maxwell equations in microcosm (the Maxwell equation for only one charge) from the Maxwell equation in Macrocosm (The Maxwell equation for many many charges). Hence traditionally we started with Maxwell equation in microcosm plus the principle that field can be superimposed, and derive the Maxwell equations of macrocosm. However from above discussion we have now clear that the superimposition principle has problem. We have give up the field can be superimposed this concept. Hence even we have the Maxwell equations in microcosm can be principle, they still cannot derive the Maxwell equation in macrocosm. Without a Maxwell equation in macrocosm, one with a Maxwell equation in microcosm we still cannot build the whole theory of electromagnetic field theory. In other hand for mutual energy theorem, the macrocosm mutual energy theorem just the summation of all the mutual energy theorem for a pair charges. If the action and reaction of a pair charges defined by the mutual energy theorem, the whole mutual energy is just found all connection pairs and put all this together. Hence from microcosm to the macrocosm for the mutual energy theorem is easy to do.

K. The field can only be defined as a approximate concept in case $N \rightarrow \infty$

We assume in electromagnetic field, Maxwell equations are not exactly correct. The field superimposed principle is also not exactly satisfied. That means we cannot have

$$\vec{E} = \sum_{i=1}^N \vec{E}_i \quad (201)$$

What we have is the only mutual energy principle. In a two electrons' system the mutual energy principle tell us

$$\begin{aligned}
& -\nabla \cdot (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \\
& = \vec{E}_2 \cdot \vec{J}_1 + \vec{E}_1 \cdot \vec{J}_2
\end{aligned}$$

$$+\vec{E}_1 \cdot \partial \vec{D}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{H}_2 \cdot \partial \vec{B}_1 \quad (202)$$

This is the starting point. This is the principle of the whole electromagnetic field theory. If there are more charges, the mutual energy principle becomes,

$$\begin{aligned}
& -\nabla \cdot (\sum_j \sum_{i \neq j} \vec{E}_i \times \vec{H}_j) \\
& = \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j) \\
& + \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)
\end{aligned} \quad (203)$$

where

$$i, j = 1, 2 \cdots N \quad (204)$$

When $N \rightarrow \infty$, the following formula is close to the above one, hence is also correct approximately,

$$\begin{aligned}
& -\nabla \cdot (\sum_j \sum_i \vec{E}_i \times \vec{H}_j) \\
& = \sum_j \sum_i (\vec{E}_i \cdot \vec{J}_j) \\
& + \sum_j \sum_i (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)
\end{aligned} \quad (205)$$

or

$$\begin{aligned}
& -\nabla \cdot (\sum_i \vec{E}_i \times \sum_j \vec{H}_j) \\
& = (\sum_i \vec{E}_i \cdot \sum_j \vec{J}_j)
\end{aligned}$$

$$+(\sum_i \vec{E}_i \cdot \partial \sum_j \vec{D}_j + \sum_i \vec{H}_i \cdot \partial \sum_j \vec{B}_j) \quad (206)$$

or

$$-\nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B} \quad (207)$$

This is the Poynting theorem. We have written

$$\vec{E} = \sum_i \vec{E}_i \quad (208)$$

,

$$\vec{H} = \sum_i \vec{H}_i \quad (209)$$

,

$$\vec{J} = \sum_i \vec{J}_i \quad (210)$$

and

$$\vec{D} = \sum_i \vec{D}_i \quad (211)$$

,

$$\vec{B} = \sum_i \vec{B}_i \quad (212)$$

Hence we obtained the results if $N \rightarrow \infty$ we have the Poynting theorem.

Important to know that here Poynting theorem is not a exact correct physics formula, but only when $N \rightarrow \infty$ it is correct approximatively. From Poynting theorem we can have

$$0 = \nabla \times \vec{E} \cdot \vec{H} - \nabla \times \vec{H} \cdot \vec{E} + \vec{E} \cdot \vec{J} + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B} \quad (213)$$

or

$$(\nabla \times \vec{E} + \partial \vec{B}) \cdot \vec{H} + (-\nabla \times \vec{H} + \vec{J} + \partial \vec{D}) \cdot \vec{E} = 0 \quad (214)$$

We obtained the sufficient conditions

$$\nabla \times \vec{E} + \partial \vec{B} = 0 \quad (215)$$

$$-\nabla \times \vec{H} + \vec{J} + \partial \vec{D} = 0 \quad (216)$$

or

$$\nabla \times \vec{E} = -\partial \vec{B} \quad (217)$$

$$\nabla \times \vec{H} = \vec{J} + \partial \vec{D} \quad (218)$$

These are Maxwell equations. Here we did not derived the Maxwell equation as the sufficient and necessary conditions. However since Poynting theorem can derive all reciprocity theorems, from the reciprocity theorems we can get all Green function solution of Maxwell equations. If we have obtained all solution of Maxwell equations we can obtained Maxwell equations by induction. Hence we finally obtained the Maxwell equations.

Please keep in mind that from the authors' above theory, that the Poynting theorem and Maxwell equations all are only approximatively correct. It only correct when $N \rightarrow \infty$.

In this way the Maxwell equations are simplified to only with one formula. Two formula be come one mutual energy principle formula. However we are not only try to simplify the Maxwell equations. We found that Maxwell equation is wrong. It need to be correct. It is only correct at $N \rightarrow \infty$. It is possible has huge problem even $N \rightarrow \infty$. That is problem the infinity problem the quantum physics meet, there a re-normalization process is required.

We achieved this result by many deep thought. The following we will offers how we finally achieved the above result. In the following we will shown in the beginning we did not know the problem is at Maxwell equations.

L. The other two Maxwell equations

The other two equations of Maxwell equations are,

$$\nabla \cdot \vec{B} = 0 \quad (219)$$

$$\nabla \cdot \vec{D} = \rho \quad (220)$$

or in the integral formula,

$$\oint_{\Gamma} \vec{B} \cdot \hat{n} d\Gamma = 0 \quad (221)$$

or

$$\oint_{\Gamma} \vec{D} \cdot \hat{n} d\Gamma = \iiint_V \rho dV \quad (222)$$

The last formula is derive from Coulomb's law which is

$$\vec{F}_{ji} = \frac{q_i q_j}{4\pi\epsilon_0} \frac{\|\vec{x}_i - \vec{x}_j\|}{\|\vec{x}_i - \vec{x}_j\|^3} \quad (223)$$

$$\vec{E}_{ji} = \frac{\vec{F}_{ji}}{q_i} = \frac{q_j}{4\pi\epsilon_0} \frac{\|\vec{x}_i - \vec{x}_j\|}{\|\vec{x}_i - \vec{x}_j\|^3} \quad (224)$$

or \vec{E}_{ji} is the charge q_j produced field at the position \vec{x}_i . The total force on the charge q_i is

$$F_i = \sum_{j=1, j \neq i}^{j=N} \vec{F}_{ji} = \sum_{j=1, j \neq i}^{j=N} \vec{E}_{ji} q_i \quad (225)$$

The total power is

$$P = \sum_{i=1}^N F_i v_i = \sum_{i=1}^N \sum_{j=1, j \neq i}^{j=N} \vec{E}_{ji} q_i v_i \quad (226)$$

This power should be vanishes

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^{j=N} \vec{E}_{ji} \vec{J}_i = 0 \quad (227)$$

where $J = q_i v_i$ we find this is also the mutual energy theorem in static situation. In the case there is no radiation filed hence in the mutual energy theorem the term

$$-\nabla \cdot \left(\sum_i \sum_{j \neq i} \vec{E}_i \times \vec{H}_j \right) \quad (228)$$

and

$$+ \sum_i \sum_{j \neq i} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \quad (229)$$

are all vanish. Hence this formula can also merge into the mutual energy theorem. The formula is not very important. Even in Maxwell theory it can only be applied to find a constant. In Maxwell theory

$$\nabla \times \vec{E} + \partial \vec{B} = 0 \quad (230)$$

$$\nabla \cdot (\nabla \times \vec{E} + \partial \vec{B}) = 0 \quad (231)$$

or

$$\nabla \cdot \nabla \times \vec{E} + \partial \nabla \cdot \vec{B} = 0 \quad (232)$$

or

$$\partial \nabla \cdot \vec{B} = 0 \quad (233)$$

or

$$\nabla \cdot \vec{B} = \text{Constant} \quad (234)$$

Because we thought the field any way is a problematic, we are not care a constant field. Hence for this equation we do not need to replace it.

XIII. RECONSTRUCTION THE ELECTROMAGNETIC FIELD THEORY FROM THE MUTUAL ENERGY PRINCIPLE

A. Define the multiplication of fields

We do not define the superimposition of the field. We do not know how to “add” is really correct. However we can redefine the “ \times ”, “ \cdot ”

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{A}_j \vec{B}_i \quad (235)$$

$$\vec{A} \times \vec{B} = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{A}_j \vec{B}_i \quad (236)$$

In this way the mutual energy principle

$$\begin{aligned} & -\nabla \cdot \left(\sum_i \sum_{j \neq i} \vec{E}_i \times \vec{H}_j \right) \\ & = \sum_i \sum_{j \neq i} (\vec{E}_i \cdot \vec{J}_j) \\ & + \sum_i \sum_{j \neq i} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \end{aligned} \quad (237)$$

can be written as

$$-\nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B} \quad (238)$$

This looks exactly to the Poynting theorem. But the above actually is the mutual energy principle, through redefine the cross multiplication and point multiplication, it can be written as the exact form of Poynting theorem. It is clear that

$$\lim_{N \rightarrow \infty} (\times_{new}) = \times \quad (239)$$

$$\lim_{N \rightarrow \infty} (\cdot_{new}) = \cdot \quad (240)$$

This guarantee that the limit situation of the mutual energy theorem is the Poynting theorem.

If Poynting theorem is established, then the superimposition of the field also can be established. Hence if $N \rightarrow \infty$ there is

$$E = \sum_{j=1}^N E_j \quad (241)$$

seems can be accept.

B. Linearization of electromagnetic fields

The fields is not linear, that is not very convenient. In order to make things simple, We can add self energy current to the mutual energy theory:

Even we known that self energy current formula

$$-\nabla \cdot \vec{E}_i \times \vec{H}_i = \vec{E}_i \cdot \vec{J}_i + \vec{E}_i \cdot \partial \vec{D}_i + \vec{H}_i \cdot \partial \vec{B}_i \quad (242)$$

is nonsense in physics, in physics, actually that $\vec{E}_i \times \vec{H}_i$ is 0, $\vec{E}_i \cdot \vec{J}_i$ is 0, but the mathematics calculation that is not 0. We also know that $\vec{E}_i = \infty$, if the emitter or the absorber is point, we can just assume it is not a point but the charge of the electron is inside a small sphere region. Then the above formula can be a correct formula in mathematics. It is a pseudo self energy current formula. We can add this pseudo formula to the following mutual energy theorem,

$$\begin{aligned}
& -\nabla \cdot \left(\sum_j \sum_{i \neq j} \vec{E}_i \times \vec{H}_j \right) \\
& = \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \vec{J}_j) \\
& + \sum_j \sum_{i \neq j} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)
\end{aligned} \tag{243}$$

We got

$$\begin{aligned}
& -\nabla \cdot \left(\sum_j \sum_i \vec{E}_i \times \vec{H}_j \right) \\
& = \sum_j \sum_i (\vec{E}_i \cdot \vec{J}_j) \\
& + \sum_j \sum_i (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j)
\end{aligned} \tag{244}$$

Keep in mind the above formula is only correct in the meaning of mathematics, not physics.

It is a mathematical formula not a physic formula. The formula can be further written as

$$\begin{aligned}
& -\nabla \cdot \left(\sum_i \vec{E}_i \right) \times \left(\sum_j \vec{H}_j \right) \\
& = \left(\sum_i \vec{E}_i \right) \cdot \left(\sum_j \vec{J}_j \right) \\
& + \left(\sum_i \vec{E}_i \right) \cdot \left(\sum_j \partial \vec{D}_j \right) + \left(\sum_i \vec{H}_i \right) \cdot \left(\sum_j \partial \vec{B}_j \right)
\end{aligned} \tag{245}$$

Write

$$\vec{E} = \sum_i \vec{E}_i, \quad \vec{H} = \sum_i \vec{H}_i, \quad \vec{J} = \sum_i \vec{J}_i \tag{246}$$

We obtain that,

$$\begin{aligned}
& -\nabla \cdot \vec{E} \times \vec{H} \\
& = \vec{E} \cdot \vec{J} \\
& + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}
\end{aligned} \tag{247}$$

Hence we obtained the Poynting theorem. We have obtain the Poynting theorem. We also obtains the new field definition, which can be superimposed. Keep in mind this all mathematical result not a results in physics. It is a result when we add pseudo self-energy current to the mutual energy principle.

The Poynting theorem can be written as

$$\begin{aligned}
& -(\nabla \times \vec{E} \cdot \vec{H} - \nabla \times \vec{H} \cdot \vec{E}) \\
& = \vec{E} \cdot \vec{J} \\
& + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}
\end{aligned} \tag{248}$$

or

$$-(\nabla \times \vec{E} + \partial \vec{B}) \cdot \vec{H} + (\nabla \times \vec{H} - \vec{J} - \partial \vec{D}) \cdot \vec{E} = 0 \tag{249}$$

The sufficient condition of the above formula is

$$\nabla \times \vec{E} + \partial \vec{B} = 0 \tag{250}$$

$$\nabla \times \vec{H} - \vec{J} - \partial \vec{D} = 0 \tag{251}$$

We got the Maxwell equation. We did not get it as a necessarily condition. But this is enough. In our electromagnetic theory, Maxwell equation is not need to be derived, it has problem anyway. The above derivation of the Maxwell equation is also by dint of the pseudo self-energy current.

Hence we obtained the results, started from mutual energy energy principle, by dint of pseudo self-energy current we prove that the Poynting theorem and Maxwell equation are still correct in the mathematics. It is notice that it only correct on mathematics not on the physics.

Understand that we can now further why in quantum physics need a re-normalization process, when it take away all self energy terms in the formula, they got correct result. In the correct physics these all self energy terms actual should be take away from the correct physics or the view of the mutual energy principle.

XIV. WIRELESS WAVE EQUATIONS

We have to define what is the wireless wave. We speak the wireless wave means this wave is send out continuously not as wave package. We known that the wireless wave frequency is very low the wave length is wave long.

In the light situation the assume one photon package has energy

$$E_{engergy} = \omega \hbar \quad (252)$$

The speed of photon is c , assume the photon has the life time Δt

$$pc = fh \quad (253)$$

or

$$\lambda = \frac{c}{f} = \frac{h}{p} \quad (254)$$

If photon become a wave, it need at least one wave length. If the wireless wave is very long for example one meter. If the wave is even continues, the surrounding can have thousand environment receiving device synchronized with this antenna. They all can receive the energy. This is different to the light situation, in which the wave can only be received by one absorber. The life time of the wave is

$$T_{life} = \frac{1}{f} \quad (255)$$

Hence if the higher the frequency, the shorter the lift time of the photon. We known that photon is composed with retarded wave and advanced wave, the two wave need to be synchronized. The above formula tell use when the frequency become higher, it is more difficult to make the two photon synchronized. When frequency is higher energy package become larger, in the light source there cannot offer continual wave, hence they can only send wave package. This wave package of retarded wave looking a synchronized advanced wave. Since the synchronization become so difficult, it can only find one to be synchronized. Hence we got photon, it is a synchronized wave from a pair electrons, one is the emitter another is the absorber. Here we know the synchronization means the time window, the frequency window and the orientation window (similar to the receive antenna, which need to adjust its direction receive more energy).

In the lower frequency, in case one energy package can be received by many many particle, it be come wireless wave.

We will show that, for photon, there is only wave between two electrons emitter and the absorber, in this situation the wave do not satisfy the Maxwell equation in macrocosm. For wireless wave if it satisfy Maxwell equations in macrocosm, that means it is not send as photon. The energy of wireless wave is one emitter corresponding to many absorbers. This kind of wave is not photon wave.

A. Mutual energy principle can be separable

Here we speak about the separation. That means for example if there are N charges in a system, we can divided this charges as 2 groups, then the mutual energy theorem can be established for this two groups. In this mutual energy theorem for groups, all interaction inside the groups will not need to be counted.

The second example for example a charge, we do not need the charge is with 0 radio. The charge can have radio and can has a few pars. For example we can divided the a electron's charge to two parts, part 1 and part 2. There is the mutual energy between this two parts. But this two parts can be see as a group of this two parts. When we calculate the field can see this group as a whole body. We do not care the mutual energy or direct interaction between the two parts inside the group.

Hence there is still need some external force to glue the all parts of the charges inside a electron. But in most situation the electron can be seen as whole and direct reaction with other electrons.

B. Why retarded antenna can be see send only retarded waves?

According to the mutual energy theorem the transmitter antenna send retarded wave and the receive antenna send advanced wave. We make a very simple example, in this example the transmitter antenna is sit at the original point of the world coordinates. Assume there are infinite more receiving antenna are at the big sphere. The center of sphere is the transmitting antenna. For the whole system we the mutual energy theorem,

$$- \oint_{\Gamma} \cdot \left(\sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}_i \times \vec{H}_j \right) \cdot \hat{n} d\Gamma$$

$$\begin{aligned}
&= \iiint_V \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\vec{E}_i \cdot \vec{J}_j) dV \\
&+ \iiint_V \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) dV
\end{aligned}$$

In the above formula, we assume for example $i = 1$ is the transmitter antenna send retarded wave. All other from $i = 2$ to N are all receive antenna stayed at the big sphere. We know we can add a pseudo self items to keep the above formula still correct in mathematics, so we have,

$$\begin{aligned}
&- \oint_{\Gamma} \cdot (\sum_{i=1}^N \sum_{j=1}^N \vec{E}_i \times \vec{H}_j) \cdot \hat{n} d\Gamma \\
&= \iiint_V \sum_{i=1}^N \sum_{j=1}^N (\vec{E}_i \cdot \vec{J}_j) dV \\
&+ \iiint_V \sum_{i=1}^N \sum_{j=1}^N (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) dV
\end{aligned}$$

or

$$\begin{aligned}
&- \oint_{\Gamma} \cdot (\sum_{i=1}^N \vec{E}_i) \times (\sum_{j=1}^N \vec{H}_j) \cdot \hat{n} d\Gamma \\
&= \iiint_V (\sum_{i=1}^N \vec{E}_i) \cdot (\sum_{j=1}^N \vec{J}_j) dV \\
&+ \iiint_V (\sum_{i=1}^N \vec{E}_i) \cdot \sum_{j=1}^N \partial \vec{D}_j + (\sum_{i=1}^N \vec{H}_i) \cdot (\sum_{j=1}^N \partial \vec{B}_j) dV
\end{aligned}$$

or

$$\begin{aligned}
&- \oint_{\Gamma} \cdot \vec{E} \times \vec{H} \cdot \hat{n} d\Gamma \\
&= \iiint_V \vec{E} \cdot \vec{J} dV \\
&+ \iiint_V (\vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}) dV
\end{aligned}$$

If we take the volume not only for V_1 , the above formula can be written as,

$$\begin{aligned}
& - \oint_{\Gamma_1} \vec{E} \times \vec{H} \cdot \hat{n} d\Gamma \\
& = \iiint_{V_1} \vec{E} \cdot \vec{J}_1 dV \\
& + \iiint_{V_1} (\vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}) dV
\end{aligned}$$

Here Γ_1 is the surface only includes the antenna 1 or the volume V_1 . Here \vec{J}_1 is the current of the transmitter antenna. \vec{E} and \vec{H} are the total field include the field of the transmitting antenna which is the retarded field and all the field of the receiving antenna which is advanced field. This become all tradition meanings in which the transmuting antenna only send the retarded wave.

To all normal people we can just see the whole field is produced by the current of the transmitting antenna.

This is the Poynting theorem in our normal sense, in which the field is all field we can measured which should include the contribution of the retarded fields and advanced fields, and also the pseudo fields. \vec{J}_1 is only the current of transmitting antenna, all the currents of receiving antenna is not list out, same as our traditional way to work with electromagnetic field. In this situation we can think our the field is produced by current of the emitter. In this way we have the felling that all the field si produced by the antenna 1 and all the field is retarded fields.

The field satisfy the above Poynting theorem guarantees it also satisfy the Maxwell equations. We have said that any field satisfy Poynting theorem, it will also satisfy the reciprocity theorem and also Green function theory, which gives all solution of the Poynting theorem, from that we obtains the Maxwell equation by induction. Hence the above field also satisfies,

$$\begin{cases} \nabla \times \vec{E} = -\partial \vec{B} \\ \nabla \times \vec{H} = \vec{J} + \partial \vec{D} \end{cases}$$

Where

$$\vec{E} = \vec{E}_1 + \dots \vec{E}_i \dots + \vec{E}_N$$

$$\vec{H} = \vec{H}_1 + \dots + \vec{H}_i + \dots + \vec{H}_N$$

Please keep mind inside above field, there is pseudo self energy current terms. Hence even we obtained the Maxwell equation, it is still only correct in some mathematics meaning.

In the case need to consider the question what is the self action

$$\iiint_V \vec{E}_i \cdot \vec{J}_i dV$$

self energy current

$$\oint_{\Gamma_1} \vec{E}_i \times \vec{H}_i \cdot \hat{n} d\Gamma$$

and what is self energy

$$\iiint_V (\vec{E}_i \cdot \partial \vec{D}_i + \vec{H}_i \cdot \partial \vec{B}_i) dV$$

The solution of Maxwell equation will meet big problems. We should be clear that the above all self energy terms are all not exist in the physics. They all illusive.

When I was study electromagnetic field theory. One of my teacher tell me that the reciprocity theorem is very strong. It can solve nearly all electromagnetic problems. In that time I asked myself that is the reciprocity theorem can replace the Maxwell equations? Now I know the reason. The reciprocity theorem is a transform of the mutual energy theorem or the mutual energy principle, which can solve all problems of electromagnetic fields.

C. The influence of the environment to the transmitting antenna

We have noticed that the field of the transmitting antenna has also the contribution from all receiving antenna which is actually the environment of the transmuting antenna. It is clear if this environment is not equal in all direction our calculated field will have big difference with the situation when we calculate it as retarded wave alone. As a antenna engineer I know the directivity diagram of the transmitting antenna is always deviate from the calculation, we often interpreted this is because the refraction of the environment. From the discussion of this section we known that this deviation perhaps is because the influence of the advanced wave from the environment.

In the next section we will assume the transmitting antenna only send the the retarded wave and we do not need to take the contribution of the advanced wave of the receiving antenna, which offers us a field of from the environment.

D. Mutual energy theorem can be applied to a part of system

For example we have a antenna is contains N electrons running inside a wire. Assume this N electron send retarded potential to outside. We would like to calculate the whole energy of this system to the outside. We know that the power of the system is

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N (\vec{E}_i \cdot \vec{J}_j) \quad (256)$$

we can calculate all the energy current go to outside space while is

$$-\nabla \cdot \left(\sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}_i \times \vec{H}_j \right) \quad (257)$$

The energy saved to the space is

$$\sum_i \sum_{j \neq i} (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \quad (258)$$

When N is very big, i.e., $N \rightarrow \infty$ the above energy go to out side have very small difference to

$$\begin{aligned} & -\nabla \cdot \left(\sum_{i=1}^N \sum_{j=1}^N \vec{E}_i \times \vec{H}_j \right) \\ &= \sum_{i=1}^N \sum_{j=1}^N (\vec{E}_i \cdot \vec{J}_j) \\ &+ \sum_{i=1}^N \sum_{j=1}^N (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) \end{aligned} \quad (259)$$

or

$$\begin{aligned} & -\nabla \cdot \left(\sum_{i=1}^N \vec{E}_i \right) \times \left(\sum_{j=1}^N \vec{H}_j \right) \\ &= \left(\sum_{i=1}^N \vec{E}_i \right) \cdot \left(\sum_{j=1}^N \vec{J}_j \right) \\ &+ \left(\sum_{i=1}^N \vec{E}_i \right) \cdot \left(\sum_{j=1}^N \partial \vec{D}_j \right) + \left(\sum_{i=1}^N \vec{H}_i \right) \cdot \left(\sum_{j=1}^N \partial \vec{B}_j \right) \end{aligned} \quad (260)$$

or

$$\begin{aligned}
& -\nabla \cdot \vec{E} \times \vec{H} \\
& = \vec{E} \cdot \vec{H} \\
& + \vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}
\end{aligned} \tag{261}$$

This is just the Poynting theorem, hence Poynting theorem still can be applied approximately.

E. Example of 3 charges or 3 antennas

Think a example of antenna system there are one transmitting antenna 1, two receiving antenna 2 and 3 in the whole space. Assume the two receive antenna is very close to each other.

The energy sent out from the transmitting antenna 1 is received by antenna 2 and 3. The transmitting antenna sent retarded potential, the two receiving antenna sent advanced potentials.

From the mutual energy principle, it tells us that

$$\begin{aligned}
& - \oint_{\Gamma} \cdot \left(\sum_{i=1}^3 \sum_{j=1, j \neq i}^3 \vec{E}_i \times \vec{H}_j \right) \cdot \hat{n} d\Gamma \\
& = \iiint_V \sum_{i=1}^3 \sum_{j=1, j \neq i}^3 (\vec{E}_i \cdot \vec{J}_j) dV \\
& + \iiint_V \sum_{i=1}^3 \sum_{j=1, j \neq i}^3 (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) dV
\end{aligned} \tag{262}$$

In another side, the mutual energy theorem is also true to each pair of two antenna. That means there is also, for example the antenna 1 and 2 have the following mutual energy theorem,

$$- \oint_{\Gamma} (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} d\Gamma$$

$$\begin{aligned}
&= \iiint_V (\vec{E}_1 \cdot \vec{J}_2 + \vec{E}_2 \cdot \vec{J}_1) dV \\
&\iiint_V (\vec{E}_1 \cdot \partial \vec{D}_2 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_2 \cdot \partial \vec{B}_1) dV
\end{aligned} \tag{263}$$

Considering antenna 1 send retarded wave and antenna 2 send the advanced wave. Hence, in the big sphere surface Γ , the field ζ_1 and field ζ_2 are not nonzero in the same time hence

$$\oint_{\Gamma} (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} d\Gamma = 0 \tag{264}$$

Here Γ is the surface contained the antenna 1 and antenna 2.

If considering a integral with a time

$$\int_{-\infty}^{\infty} \iiint_V (\vec{E}_1 \cdot \partial \vec{D}_2 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_2 \cdot \partial \vec{B}_1) dV dt = 0 \tag{265}$$

hence we have

$$\int_{-\infty}^{\infty} \iiint_V (\vec{E}_1 \cdot \vec{J}_2 + \vec{E}_2 \cdot \vec{J}_1) dV dt = 0 \tag{266}$$

or

$$-\int_{-\infty}^{\infty} \iiint_V \vec{E}_1 \cdot \vec{J}_2 dV dt = \int_{-\infty}^{\infty} \iiint_V \vec{E}_2 \cdot \vec{J}_1 dV dt \tag{267}$$

The left side is the energy sent by the transmitting antenna. The right side is the energy received by the second antenna.

In the same way we have

$$-\int_{-\infty}^{\infty} \iiint_V \vec{E}_1 \cdot \vec{J}_3 dV dt = \int_{-\infty}^{\infty} \iiint_V \vec{E}_3 \cdot \vec{J}_1 dV dt \tag{268}$$

The left is the energy sent by the first antenna to the second antenna. The second is the energy received by the third antenna.

If the antenna 2 and 3 is very close, they can have influence by each other. This can be described by the mutual energy theorem between 2 and 3. Since these two antenna are all receiving antenna. They sent all advanced wave. in this situation the surface integral is nonzero, hence we have

$$\int_{-\infty}^{\infty} \iiint_V (\vec{E}_2 \cdot \vec{J}_3 + \vec{E}_3 \cdot \vec{J}_2) dV dt = \int_{-\infty}^{\infty} \oint_{\Gamma} (\vec{E}_2 \times \vec{H}_3 + \vec{E}_3 \times \vec{H}_2) \cdot \hat{n} d\Gamma \quad (269)$$

The two antenna will enforce the receiving strength make two receiving antenna receive power more than the summation of the two antenna worked alone. This become more complicated we do not continually discuss here.

It is same if there two transmitting antenna close to each other, they can send energy more than two 2 times as the antenna worked alone.

We can add the pseudo self energy terms to the mutual energy theorem, hence we obtained,

$$\begin{aligned} & - \oint_{\Gamma} \cdot \left(\sum_i^3 \sum_{j=1}^3 \vec{E}_i \times \vec{H}_j \right) \cdot \hat{n} d\Gamma \\ & = \iiint_V \sum_{i=1}^3 \sum_{j=1}^3 (\vec{E}_i \cdot \vec{J}_j) dV \\ & + \iiint_V \sum_{i=1}^3 \sum_{j=1}^3 (\vec{E}_i \cdot \partial \vec{D}_j + \vec{H}_i \cdot \partial \vec{B}_j) dV \end{aligned} \quad (270)$$

or

$$\begin{aligned} & - \oint_{\Gamma} \cdot \left(\sum_i^3 \vec{E}_i \right) \times \left(\sum_{j=1}^3 \vec{H}_j \right) \cdot \hat{n} d\Gamma \\ & = \iiint_V \left(\sum_{i=1}^3 \vec{E}_i \right) \cdot \left(\sum_{j=1}^3 \vec{J}_j \right) dV \\ & + \iiint_V \left(\sum_{i=1}^3 \vec{E}_i \right) \cdot \sum_{j=1}^3 \partial \vec{D}_j + \left(\sum_{i=1}^3 \vec{H}_i \right) \cdot \sum_{j=1}^3 \partial \vec{B}_j dV \end{aligned} \quad (271)$$

or

$$\begin{aligned} & - \oint_{\Gamma} \cdot \vec{E} \times \vec{H} \cdot \hat{n} d\Gamma \\ & = \iiint_V \vec{E} \cdot \vec{J} dV \end{aligned}$$

$$+ \iiint_V (\vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}) dV \quad (272)$$

if we take the volume not only for V_1 , the above formula can be written as,

$$\begin{aligned} & - \oint_{\Gamma_1} \vec{E} \times \vec{H} \cdot \hat{n} d\Gamma \\ & = \iiint_{V_1} \vec{E} \cdot \vec{J}_1 dV \\ & + \iiint_{V_1} (\vec{E} \cdot \partial \vec{D} + \vec{H} \cdot \partial \vec{B}) dV \end{aligned} \quad (273)$$

This is the Poynting theorem in our normal sense, in which the field is all field we can measured which should include the contribution of the retarded fields and advanced fields, and also the pseudo fields. \vec{J}_1 is only the current of transmitting antenna, all the currents of receiving antenna is not list out, same as our traditional way to work with electromagnetic field. In this situation we can think our the field is produced by current of the emitter. In this way we have the felling that all the field si produced by the antenna 1 and all the field is retarded fields.

The field satisfy the above Poynting theorem guarantees it also satisfy the Maxwell equations. We have said that any field satisfy Poynting theorem, it will also satisfy the reciprocity theorem and also Green function theory, which gives all solution of the Poynting theorem, from that we obtains the Maxwell equation by induction. Hence the above field also satisfies,

$$\begin{cases} \nabla \times \vec{E} = -\partial \vec{B} \\ \nabla \times \vec{H} = +\vec{J} + \partial \vec{D} \end{cases} \quad (274)$$

Where

$$\begin{aligned} \vec{E} &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \\ \vec{H} &= \vec{H}_1 + \vec{H}_2 + \vec{H}_3 \end{aligned} \quad (275)$$

Please keep mind inside above field, there is pseudo self energy current terms. Hence even we obtained the Maxwell equation, it is still only correct in some mathematics meaning.

In the case need to consider the question what is the self action

$$\iiint_V \vec{E}_i \cdot \vec{J}_i dV \quad (276)$$

, self energy current

$$\oint_{\Gamma_1} \vec{E}_i \times \vec{H}_i \cdot \hat{n} d\Gamma \quad (277)$$

and what is self energy

$$\iiint_V (\vec{E}_i \cdot \partial \vec{D}_i + \vec{H}_i \cdot \partial \vec{B}_i) dV \quad (278)$$

The solution of Maxwell equation will meet big problems. We should be clear that the above all self energy tems are all not exist in the physics. They all illusive.

When I was study electromagnetic field theory. One of my teacher tell me that the reciprocity theorem is very strong. It can solve nearly all electromagnetic problems. In that time I asked myself that is the reciprocity theorem can replace the Maxwell equations? Now I know the reason. The reciprocity theorem is a transform of the mutual energy theorem or the mutual energy principle, which can solve all problems of electromagnetic fields.

XV. PHOTON EQUATIONS

A. The mutual energy theorem

Now I am cleared that I am actually do not need to calculate the self energy current, which doesn't exist.

The reason, the self energy current doesn't exist is because, the concept electromagnetic field is wrong. The electromagnetic field need a test charge in static field situation or an absorber in light wave situation to measure the field or absorb the field. However this charge or absorber actually joined the creation of the action or reaction. We have measured the field we cannot show whether or not if the test charges or the absorber is removed from the system the electromagnetic field still exist. According to the direct interaction principle, this action or reaction is only exist in the case there are two electrons, the emitter and the absorber.

This problem cannot solved through the concept field. But we have successfully solved it through the energy. From the Poynting theorem we remove all self items include self energy

current, self energy increase and self reaction ($\vec{J}_i \cdot \vec{E}_i$). After this removal, the Poynting theorem is changed to the mutual energy theorem.

If self energy current doesn't exist. The Maxwell equation is clear wrong, because from Maxwell equations we can get a solution with self energy current. It even worse, we can not get a solution with Maxwell equation that self energy vanishes. Hence, up to now the only way is to abandon the Maxwell equations. Take self energy current away and Maxwell equation away, the left is only the mutual energy theorem and we can call it as mutual energy principle. The above few sections we have shown that the mutual energy can be used to replace Poynting theorem, after this replacement, we got a new "Poynting theorem" which is actually the mutual energy theorem. We also shows the Poynting theorem is equivalent in principle to the Maxwell equations. Hence when we can replace the Poynting theorem with the mutual energy theorem, the mutual energy theorem actually can also replace the Maxwell equations. After we also shows that the Gauss law can also merged to the mutual energy theorem, Hence we can use mutual energy theorem only one formula to replace all 4 formula of the Maxwell equations. This replacement is not because 1 formula is simpler than 4 formula, it is because this one formula is correct and the 4 formula is not correct.

We also shows the problem of the Maxwell equation in macrocosm, if we add a pseudo field to the two sides of the formula of the mutual energy theorem, it become the Poynting theorem, and the Poynting theorem is still correct in mathematics. Hence we also can show the Maxwell equations are also correct in mathematics. It is not correct in physics, we have notice there are pseudo field items. However, most wireless problem which still can be solved with Maxwell equations. Here the only need to notice is that the concept of the field is only correct at adding the pseudo items, which is all self energy items. For most wireless situation we still can apply all engineer problem with Maxwell equations.

In the case of light, there is only two charges, one is emitter and one is absorber. This is the place we really need to deal the problem of self energy items. We have calculated that if it exist, it will contribute half energy transfer from emitter to absorber. In this situation the self energy items cannot be omitted. We endorse the direct interaction principle, which leads us to denies all existent of the self energy items. After removal of all self energy items, we obtained the mutual energy theorem or the mutual energy principle. Now we need to looking the solution from the mutual energy principle.

Hence, this means that we only need to find a solution which satisfy the mutual energy

principle. For the photon situation there is only the emitter and absorber two electron, we assume the index of the emitter is 1 and the index of the absorber is 2, the mutual energy theorem is list as following,

$$\begin{aligned}
& -\nabla \cdot (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \\
& = \vec{E}_2 \cdot \vec{J}_1 + \vec{E}_1 \cdot \vec{J}_2 \\
& + \vec{E}_1 \cdot \partial \vec{D}_2 + \vec{E}_2 \cdot \partial \vec{D}_1 + \vec{H}_1 \cdot \partial \vec{B}_2 + \vec{H}_2 \cdot \partial \vec{B}_1
\end{aligned} \tag{279}$$

B. The solution of photon equations 1

We know the Maxwell equations are the sufficient condition of the mutual energy theorem, hence we can get the solution of the mutual energy theorem by solving the Maxwell equations. One of the solution of the above photon equation is Maxwell equation solutions which is,

$$\begin{cases} \nabla \times \vec{E}_1 = -\partial \vec{B}_1 \\ \nabla \times \vec{H}_1 = +\vec{J}_1 + \partial \vec{D}_1 \end{cases} \tag{280}$$

and

$$\begin{cases} \nabla \times \vec{E}_2 = -\partial \vec{B}_2 \\ \nabla \times \vec{H}_2 = +\vec{J}_2 + \partial \vec{D}_2 \end{cases} \tag{281}$$

It must notice that (a) we are looking the solutions $\zeta_1 = [\vec{E}_1, \vec{H}_1, J_1]$, $\zeta_2 = [\vec{E}_2, \vec{H}_2, J_2]$ nonzero simultaneously. If $\xi_1 = [\vec{E}_1, \vec{H}_1] = 0$, $\xi_2 = [\vec{E}_2, \vec{H}_2] \neq 0$, this become the 0 solution of the mutual energy principle which is not what we are looking for. Hence the above simultaneously nonzero solution of Maxwell equations is not just the solution of the Maxwell equations but is the solution of the mutual energy principle.

Figure 2 shows the photon model of this kind solution.

Assume we have put a metal place between the emitter and the absorber. We make a hole to allow the light can go through it from the emitter to the absorber. The mutual energy is exist only on the overlap of the two field $\zeta_1 = [\vec{E}_1, \vec{H}_1]$ and $\zeta_2 = [\vec{E}_2, \vec{H}_2]$, see Figure.

The disadvantage of this photon model is that it can only send the wave with linear polarization. If we need the photon send circular polarized field, we have to make the

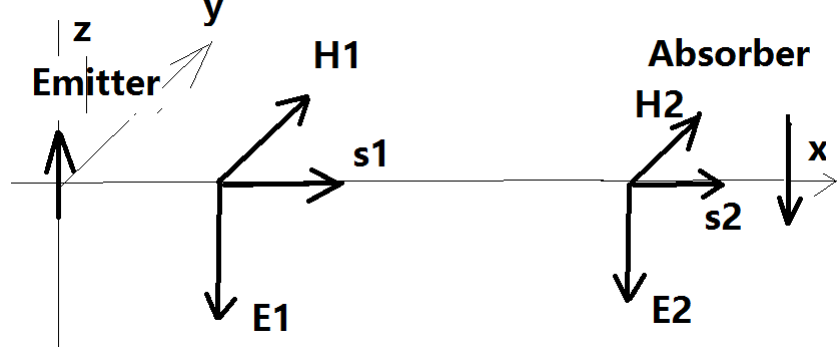


Figure 2: photon model, in this model the field $\zeta_1 = [\vec{E}_1, \vec{H}_1, \vec{J}_1]$, $\zeta_2 = [\vec{E}_2, \vec{H}_2, \vec{J}_2]$ all satisfy Maxwell equations.

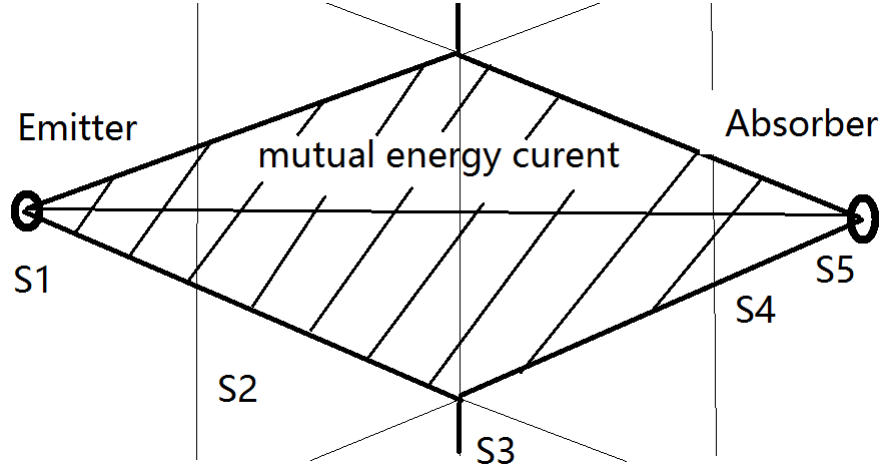


Figure 3: The mutual energy current is only existent at the overlap place of the the two solutions of the Maxwell equation. The field of the emitter is retarded wave. The wave of the absorber is advanced wave.

current \vec{J}_1 and \vec{J}_2 all have two components for example along y and z , or to make the currents rotating along x axis. This is perhaps possible, because the electron is at spin, there current is also possible to spin. In this wave the radiate wave become circular rotated.

We can take the volume only includes the emitter or only includes only the absorber, this way we can prove the mutual energy current go through each surface S_1 , S_2 , S_3 , S_4 and S_5 are equal that is

$$\begin{aligned}
 & - \int_{t=-\infty}^{\infty} \iiint_{V_1} (\vec{E}_2 \cdot \vec{J}_1) \\
 & = Q_1 = Q_2 = Q_3 = Q_4 = Q_5
 \end{aligned}$$

$$\int_{t=-\infty}^{\infty} \iiint_{V_2} (\vec{E}_1 \cdot \vec{J}_2) dV \quad (282)$$

where

$$Q_i = \int_{t=-\infty}^{\infty} \oint_{S_1} (\vec{E}_1 \times \vec{H}_2 + \vec{E}_2 \times \vec{H}_1) \cdot d\Gamma dt \quad i = 1, 2, 3, 4, 5 \quad (283)$$

This formula clear tell us the photon's energy is just the mutual energy current. The mutual energy current is equal at the 5 different surface. We know that the surface S_1 and S_2 are very close to the emitter or absorber. This time the surface be come so small hence the wave beam is concentrated to a point which look very like a particle. In the middle, the wave beam is very thick. We can put other kind plate for example the metal plate with two slits. In this case the wave will produce interference patterns. This will explain the duality character of the photon.

For the above solution, we have use the the sufficient condition of the mutual energy theorem. It is not the necessary condition. The mutual energy principle perhaps has some other solution which do not satisfy Maxwell equations, which will be discussed in next section.

C. The solution of photon equations 2

Hence this is not solution we are looking for. However, we can just rotated the 90 degree hence it will parallel to the The above self-energy current vanishes. This means $\vec{E} \times \vec{H}$ must parallel to each other.

In this situation the perhaps we can find a solution which is not the solution of Maxwell equation but it still satisfy the mutual energy principle.

If it is true this solution is also possible the solution for electromagnetic fields, that means we have found other photon model.

We can easy to prove the above Figure 4 satisfy mutual energy theorem even it is not satisfy Maxwell equations. We can see that this model is just a rotation from the Figure 2. We know that the photon model in Figure 2 satisfy Maxwell equations, that guarantees it also satisfies the mutual energy theorem. Know we have rotated \vec{J}_2 , \vec{H}_2 and \vec{E}_1 along the x axis 90 degree. Hence $\vec{E}_1 \times \vec{H}_2$ and $\vec{E}_1 \cdot \vec{J}_2$ don't change. This guaranties all the items

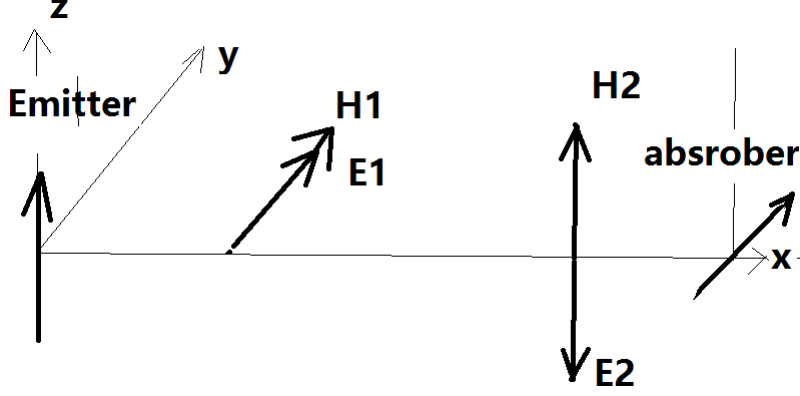


Figure 4: In this photon model, the fields $\zeta_1 = [\vec{E}_1, \vec{H}_1, \vec{J}_1], \zeta_2 = [\vec{E}_2, \vec{H}_2, \vec{J}_2]$ do not satisfy the Maxwell equations. However they still satisfy the mutual energy theorem.

in the mutual energy theorem do not change, if it is satisfied before the rotation, after the rotation the equation should still satisfied.

Even this model do not satisfy the Maxwell equations, it has 3 advantages.

(1) Since field $\vec{E}_1 \parallel \vec{H}_1$ and $\vec{E}_2 \parallel \vec{H}_2$ the self energy current vanish automatically.

$$\vec{E}_1 \parallel \vec{H}_1, \quad \vec{E}_2 \parallel \vec{H}_2 \quad (284)$$

or

$$\oint_{\Gamma} (\vec{E}_1 \times \vec{H}_1) \cdot d\Gamma = 0, \quad \oint_{\Gamma} (\vec{E}_2 \times \vec{H}_2) \cdot d\Gamma = 0 \quad (285)$$

(2) we can see that, $\vec{E}_1 \perp \vec{J}_1$ and $\vec{E}_2 \perp \vec{J}_2$ we have clear that the self energy action vanish automatically.

$$\iiint_V (\vec{E}_1 \cdot \vec{J}_1) dV = 0 \quad (286)$$

$$\iiint_V (\vec{E}_2 \cdot \vec{J}_2) dV = 0 \quad (287)$$

(3) this model are easy to support circular polarization. Because not \vec{E}_1 and \vec{E}_2 also perpendicular. We know it is difficult to explain that why electron is so small it still can send circular polarization waves. In our antenna experience we need very complicated antenna

to send circular polarized waves. When two electric field are perpendicular if the two fields have 90 degree phase difference, we have obtained circular polarization.

Even this solution do not satisfy Maxwell equation, it has 0 self energy current and 0 self action and can offer a simple current to support the circular polarization. It looks as a miracle.

Even this is not a physic solution of photon, using the mutual energy theorem as principle still much better than to take the Maxwell equations as the principle.

D. Is light satisfy Maxwell equation?

Many people ask in the internet the question whether or not the field of light satisfy Maxwell equation? We need to distinguish the problem in microcosm and in macrocosm.

In microcosm deal the problem of photon, the photon is a small system only with two electrons one is the emitter, the other is the absorber. From above discussion we know that photon satisfies the mutual energy principle. We also know that the Maxwell equation in microcosm is the sufficient condition of the mutual energy principle, hence, it is possible that photon satisfies the Maxwell equations. But it is also possible photon satisfies the mutual energy principle but do not satisfy the Maxwell equations.

In macrocosm, many photo do not satisfy the Maxwell equations in macrocosm, the reason is photon is energy package, here, the energy is linear. For example one photon the energy is 1, 2 photon the energy become 2. If energy is linear, its field can not be linear. Hence in macrocosm the many photon is also do not satisfy the Maxwell equations. This result is different with wireless wave. In case of wireless wave, the wireless wave satisfy linear (do not consider the effect of pseudo field) It satisfy Maxwell equations.

According discussion that field of emitter and absorber of one photon is possible do not satisfy Maxwell equations. There is the possibility it do not satisfy Maxwell equation but it still satisfy mutual energy principle.

However to make things simple we still assume for each photon, the emitter and the absorber still satisfy the Maxwell equations. For a system with big number of photon N , there is $2N$ emitters and absorbers. If the field of the light can be superimposed, then it is clear the $2N$ system also satisfy Maxwell equations.

If

$$\begin{cases} \nabla \times \vec{E}_i = -\partial \vec{B}_i \\ \nabla \times \vec{H}_i = +\vec{J}_i + \partial \vec{D}_i \end{cases} \quad (288)$$

is satisfied it is clear that

$$\begin{cases} \nabla \times \sum_{i=1}^{2N} \vec{E}_i = -\partial \sum_{i=1}^{2N} \vec{B}_i \\ \nabla \times \sum_{i=1}^{2N} \vec{H}_i = +\vec{J}_i + \partial \sum_{i=1}^{2N} \vec{D}_i \end{cases} \quad (289)$$

However the field of photon is not linear and cannot superimposed. There are two reasons we speak the field of light is not linear. The first reason is we have mentioned before in last few sections. The second reason is photon as energy package and hence its energy is linear. N photons energy has N times of energy of one photon. Energy is linear tell us that the field is not linear. if the field is linear the energy must quadratic.

Hence in general that light's field do not satisfy Maxwell equations in macrocosm.

E. Is the wireless wave are composed as photons?

Some people ask in the internet that is the wireless wave also composed as many photons or there is frequency limit beyond that frequency all energy be come packaged as photon. We thought this is a very good question and would like make it clear here.

In the antenna system one transmitting antenna can send the energy to many receiving antennas. It is not send a package of the energy to only one antenna in a time and randomly send the the energy to another antenna. It send the energy to all space if any antenna can receiving this energy.

This is a system with charges more than two. All these receiving antennas can synchronized with the transmitting antenna with frequency and orientation. This kind of energy is not a energy package. Hence wireless wave is not composed as photons. Actually the high frequency wave become photon is only because the field of the emitter and absorber is difficult the synchronized.

F. Is all current happened in the emitter or absorber will create any action or reaction?

Assume in the emitter it has a current change for example a electron has from a higher energy level jump to a lower, is this current must send a photon? We think the answer is no. This current change will send the retarded potential but if there can not found a absorber and the field of the absorber must just synchronized to the field of the emitter. If this kind of absorber is not happened. The energy is not send out. The electron has energy perhaps it will return to the higher level. And next time it will spring to low level again to found the matched advance wave.

In the case only have a emitter and a absorber, the current of the emitter produced a retarded wave and advanced wave. The absorber as produced a retarded wave and advanced wave. We know that the retarded wave or the emitter has synchronized with the advanced wave of the absorber. But the advanced wave of the emitter and the retarded wave of the absorber has been sent out but doesn't find a corresponding matured wave.

Maxwell equation cannot tell us if there is a current advanced wave or retarded wave should be associated to this current. Hence Wheeler and Feynman assume there always half advanced wave and half retarded wave associated to the current.

From my experience I would thought only one kind wave can associated to the emitter and absorber. We can not obtained which wave this current can produce from Maxwell's theorem, but perhaps we can obtained some information from the mutual theorem.

If the charge jump from the high energy to lower energy, we can assume there are many advanced wave have been at the place of the emitter, if it just run against the advanced wave it will gave this wave as retarded potential. When this retarded potential reached to the absorber. The electron in the absorber go along the retarded wave hence it will send the advanced wave. From this reaction who should send retarded wave and who should send advance wave is decided.

The retarded wave is not real wave, it is only offers the possibility to support a absorber to receive it. If this absorber appear by produce a current synchronized with the emitter. The energy will send from the emitter to the absorber. If the no absorber to receive this energy. The energy of the emitter doesn't loss any energy to the empty space. There is no any energy is lost to the space and move in the space without the emitter and the absorber.

G. Retarded potential

We know that the mutual energy theorem endorse the direct interact theory, hence the retarded wave is not a real thing. If there is no absorber or the advance wave, the retarded wave cannot send the the energy out. This tell us the retarded wave is only offers the ability to send out the energy but doesn't really offers the energy. This is real reason of the probability interpretation of Copenhagen. However now I know that the probability interpretation of Copenhagen is at least correct at about the retarded wave. The retarded wave is only offers the ability to do some thing. It is not send real energy, the real energy is sent only in case there is a advanced wave.

In this way we actually endorse both the Copenhagen interpenetration of the quantum physics and also the transaction interpretation of the quantum physics in which allows the retarded wave and the advanced wave.

When I speak that the retarded wave is not real, actually we also means the advanced wave is also not real, not energy is send by advanced wave, only if the retarded wave and the advanced wave meet together, the energy current is send from emitter to the absorber.

XVI. THE STORY ABOUT THIS DISCOVERY

1984 I have enter Xidian University in China to learn electromagnetic field and microwave technology as a master degree graduate student. 1985 I begin to learn the modified reciprocity theorem from the book about electromagnetic wave of J.A Kong. In his book he has talked the modified Lorentz theorem and electromagnetic field transform, in another different section. As application I applied the transform to the reciprocity theorem I got a new formula, when I looked the new formula it is much meaningful compare to the Lorentz reciprocity theorem. It clear tells the energy between two the receiving and transmitting antennas. I call it “mutual energy theorem” and published 3 papers about that[18–20] in 1987-1989. In the end of 1986, my master degree defense is with the “mutual energy theorem” which is failed. The professors said: “make a transform to the reciprocity theorem and then call it the mutual energy theorem that is not worthy of the name”.

Later I worked as antenna designer for 3 years in China. Then go to Germany working at CT and MEG medical image in Julich research center Germany for 7 years as scientist.

This work allow me to become a Ph.D in Xi'an Jiaotong University in 1998 in the field CT image reconstructions.

Later I Changed more than 10 different companies and institute in Germany, Canada and USA, most working as C++ software research and development. My publication most is about CT image reconstructions.

From 2014 I came back to the mutual energy theorem, I have long the felling this topic I did not finish, there is some important thing I need to do. First I wrote the "Concept of mutual energy". This time I can search all reference free, because I am working at Cimtec Inc which belongs to Western University of Canada. The most important reference are two I found, one is from W.J. Welch [7, 8]. He seems first derived the embryonic form of the mutual energy theorem and introduced the concept of advanced potential. This is the first time I touched the concept of advanced potential. The second reference is the time domain mutual energy theorem published by Adrianus T. de Hoop[21]. The difference of this publication with the mutual energy theorem is only a Fourier transform. It is referred as time-domain correlation reciprocity theorem published also in 1987 seem year as the mutual energy theorem published. It is lucky for me that the publication is half years earlier than his. This allow me still can call it as the mutual energy theorem. I submitted 8 manuscripts about this mutual energy concepts to IEEE Transactions on Microwave Theory and Techniques. It is nearly go through the peer review process. They not reject it immediately but ask me to make corrections. After 3 time corrections, but finally they rejected them. I resent it to IEEE Transactions on Antennas and Propagation, they are rejected again.

In the end of 2016 I begin to realized that the mutual energy theorem is not only a theorem but is a principle. Here I did not mean it is a principle can replace Maxwell equations. I think if electromagnetic field transfers the energy by the mutual energy theorem, then other particles, for example electron will do also energy transfer by the mutual energy of the electron. In this way it is a principle.

If in wireless wave, transfer energy between transmitting antenna and receiving antenna is a combination of retarded potential and advanced potential, then the energy transfer between the emitter of the electron and the absorber of the electron also should through the mutual energy current of the electrons. In this way, the mutual energy theorem is a principle. I begin to work at to prove photon transfer the energy by the mutual energy current. I prove in a lossy media the mutual energy theorem offers correct result but the reciprocity theorem

cannot. I have tried to explain the duality of the photon with the mutual energy theorem.

In the December of 2016 I begin to work at the theory of time domain mutual energy current. I begin to interesting to know whether or not that the self-energy current do transfer the energy. In this time I am sure that the mutual energy current can transfers energy, but I am not clear what about the self-energy current. Is self energy current also transfer energy? If the self energy current transfer energy I calculated, it can have half contribution to the whole energy transfer for a system with an emitter and an absorber. Hence for photon the self energy cannot be omitted. In case N is large, the self energy have N items. The mutual energy current has $N^2 - N$ items. Each items has same energy, hence if N is very large, the contribution of self-energy current can be omitted. For a photon there is only $N = 2$, one emitter and one absorber. $2^2 - 2 = 2$. Hence the self energy current cannot omit in the case of photon.

In this time I still believe Maxwell equations. According to the Maxwell equations, if self energy current vanishes, I found that the field also will vanish.

If

$$self\ energy\ current \equiv \oint_{\Gamma} (\vec{E} \times \vec{H}) \cdot \hat{n} d\Gamma = 0 \quad (290)$$

means that

$$\iiint_V (\vec{E} \cdot \partial \vec{E} + \vec{E} \cdot \partial \vec{E}) = 0 \quad (291)$$

Here V is at the outside of the surface Γ that also further means

$$\vec{H} = \vec{E} = 0$$

However if the electromagnetic field vanishes, the mutual energy current also will vanish. This is really confused me.

In other hand, I cannot accept the concept of self energy current collapsed to absorber. Collapse is not a physics concept, if you accept the concept could you offer me the equation of the collapse process?

Assume the self energy current doesn't vanishes. I have designed a few possibility for that. The emitter's self energy current sends to future and infinity. The absorber's self energy current sends to past and infinity. The emitter lost some energy, the absorber obtained some energy hence, the self energy is transferred from emitter to the absorber.

Another possibility is that the emitter sends the retarded self energy current to the future and infinity, in the same time it sends advanced self energy current to the past and infinity, hence there is no pure energy gain for the emitter. It is same to the absorber theory.

I noticed that there is problem for these two possibility. If the source of light is put inside of the metal container, and there is only a small hole for light, how can allow the self energy go through the hole and got to infinity? Hence I believe the self energy current exist is still a very bed idea.

I begin to look the possibility the self energy current helps the mutual energy to send energy from emitter to absorber, then it returned to it's sender. The emitter's self-energy return to the emitter, the absorber's energy current returns to the absorber. However the electromagnetic theory do not support time-reversed wave. From Maxwell equations we can obtained retarded wave which is sent to future and infinity and advanced wave which is sent to the past and infinity. The returned wave for the retarded wave needs to go from future and infinity come back to the emitter. This wave actually obeys so called time-reversed Maxwell equations which is

$$\nabla \times \vec{E} = \partial \vec{B} \quad (292)$$

$$\nabla \times \vec{H} = -J - \partial \vec{D} \quad (293)$$

I have to introduce the new questions that is also not a good idea. There is also other problem, I have to ask the return process is not done immediately otherwise the returned field will cancels the original self energy current. The returned wave must stayed at infinity for a moment then returns. This is even more strange. How the wave can stay at infinity for a moment and then returns?

Hence the emitter and absorber send self-energy currents become very confuse to me. I come back to the idea the self current should vanish hence

$$\oint_{\Gamma} (\vec{E} \times \vec{H}) \cdot \hat{n} d\Gamma = 0 \quad (294)$$

if

$$\vec{E} \times \vec{H} = 0 \quad (295)$$

The above self-energy current vanishes. This means $\vec{E} \times \vec{H}$ must parallel to each other.

$$\vec{E} \parallel \vec{H} \quad (296)$$

However, Maxwell equation is clear do not support this kind of field. This parallel field cannot be the solution of the Maxwell equations. I begin to think perhaps we do not need to satisfy the Maxwell equations, we only need to find solution to satisfy

$$\nabla \times (\vec{E}_1 + \vec{E}_2) = -\partial(\vec{B}_1 + \vec{B}_2) \quad (297)$$

$$\nabla \times (\vec{H}_1 + \vec{H}_2) = (J_1 + J_2) + \partial(\vec{D}_1 + \vec{D}_2) \quad (298)$$

I have drew the picture about the field possibility. But I still could not prove this kind of field satisfy the above equations. From my experience, the new equation has doubled the variables compared to the original Maxwell equations, hence has more solutions. And should be easy to find solutions. But I still cannot cannot prove this solution satisfy this relaxed Maxwell equations. And also I cannot prove this relaxed is reasonable.

In beginning of the March in 2017, I begin to think what is correct way to define the electromagnetic fields and the magnetic field. It lead me to have 3 ways to define the field, which is the correct way? After I notice the correct power is

$$P = \sum_{i=0}^N \sum_{j=1, j \neq i}^N \vec{E}(x_j, x_i) \cdot \vec{J}_i \quad (299)$$

I begin to realized, in the total power, there is no any thing called self action, which is

$$\vec{E}(x_i, x_i) \cdot \vec{J}_i \quad (300)$$

We do not need to calculate $\vec{E}(x_i, x_i)$ that is ∞ . Quantum physics try to solve this zero infinity by re-normalization. But this formula clear tell us the self-energy action $\vec{E}(x_i, x_i) \cdot \vec{J}_i$ is a concept not necessary. If self energy current is take away from current calculation, it should be also take away from all other corresponding energy calculation. Hence all energy related value should be defined like following,

$$\vec{E}(x) \times \vec{H}(x) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}(x_j, x) \times \vec{H}(x_i, x) \quad (301)$$

$$\vec{E}(x) \cdot \vec{E}(x) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{E}(x_j, x) \cdot \vec{E}(x_i, x) \quad (302)$$

$$\vec{H}(x) \cdot \vec{H}(x) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \vec{H}(x_j, x) \cdot \vec{H}(x_i, x) \quad (303)$$

The field superimposition is a confused concept and hence can be removed. We define the field just as the collection of all contributions,

$$\vec{E}(x) = \{\vec{H}(x_1, x), \vec{H}(x_2, x), \dots \vec{H}(x_i, x) \dots\} \quad (304)$$

I am really so glad to found out the problem is at the superimposition of the field. The Maxwell equation in the microcosm is perhaps still correct, but it can also be replaced with the mutual energy principle. In the macrocosm, since the field concept is not correct, we cannot superimpose the field. Even if Maxwell equations are correct in microcosm, we still cannot prove the Maxwell equations in macrocosm is correct.

In the end I found if add the pseudo self items for the field, the Poynting theorem still correct in the meaning of the mathematics. Maxwell equations can be done the same in macrocosm. Hence all traditional way to solve the classical problem still can work with all Maxwell's theory. The only things need to be notice is that the Poynting theorem and Maxwell's equation have added the pseudo self field, which can cause problem especially in the situation of quantum physics. If we use the mutual energy theorem as a principle, all normalization process in quantum physics become clear correct.

XVII. CONCLUSION

In photon situation, Maxwell equation for a emitter or absorber, only offers a illusive solution which are nonzero solution of self energy terms. For a system of a photon with a emitter and absorber, the solution of simultaneously nonzero solution for the emitter and absorber is possible to be obtained from the mutual energy principle.

Even if we assume all the emitters and absorbers satisfy Maxwell equations in microcosm, we cannot prove the Maxwell equation in in macrocosm situation. for example wireless wave case.

That can only achieve if the field can be superimposed. However we have found in general the field cannot be superimposed. The field can be linearized, but that need to add all pseudo self energy terms. The macrocosm Maxwell equation need to be proved with mutual energy principle together the pseudo self energy terms.

In other side the mutual energy principle is established both in photon situation there is only very few source and the macrocosm situation there are infinite charges all synchronized. From mutual energy principle we can prove the Poynting theorem and Maxwell equations in macrocosm(even they are only correct in mathematics).

Hence we obtained the conclusion it should not necessary to offer the Maxwell equations as a principle. It is only a mathematics method can be used to found the solution of the mutual energy principle.

Mutual energy principle ask all field cannot vanishes. Hence the solution with mutual energy theorem will be nonzero solution for all fields. Maxwell equations can obtained the solution only for emitter or for absorber which is not the solution we are looking for. From Maxwell equations we can obtained the pseudo self energy terms which are very confuse.

Hence we have to put the mutual energy principle as the starting point of all electromagnetic theory. It is the real principle in the theory of the electromagnetic fields.

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